QUESTION PAPER CODE 65/5/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $2\ AA'| = 4|A|\ |A'|$ $\frac{1}{2}$

$= 4 \times 4 \times 4 = 64$ $\frac{1}{2}$

2. $\sin\theta = \left|\frac{(\hat{i}+\hat{j}+\hat{k})\cdot(3\hat{i}-\hat{j}+2\hat{k})}{\sqrt{1+1+1}\ \sqrt{9+1+4}}\right|$ $\frac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{4}{\sqrt{42}}\right)$ $\frac{1}{2}$

OR

Any point on $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5} = \lambda$ ...(1) $\frac{1}{2}$

is $(\lambda-2, 3\lambda+5, 5\lambda-1)$

Line (1) cuts yz plane at $\lambda - 2 = 0$ i.e., $\lambda = 2$

hence required point is (0, 11, 9) $\frac{1}{2}$

3. $\frac{dy}{dx} = 3A\sin 3x + 3B\cos 3x$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = 9A\cos 3x - 9B\sin 3x = -9y$ $\frac{1}{2}$

4. Let $y = \cos^{-1}(\sin 2x)$, then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\sin^2 2x}}(2\cos 2x)$ $\frac{1}{2}$

$= -2$ or 2 $\frac{1}{2}$

SECTION B

5. (i) $\forall\, a, b \in Z$, $a * b = 2a^2 + b \in Z$ $\therefore$ * is binary 1

(ii) $1, 2 \in Z$, $1 * 2 = 2 \times 1^2 + 2 = 4$, $2 * 1 = 2 \times (2)^2 + 1 = 9$

$1 * 2 \neq 2 * 1$

$\therefore$ * is not commutative. 1

6. $n = 4,\ p = \frac{1}{4}, q = \frac{3}{4}$ $\frac{1}{2}$

P (atleast 3 are diamonds) = P (X = 3) + P(X = 4)

$= {}^4C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right) + {}^4C_4\left(\frac{1}{4}\right)^4$ 1

$= \left(\frac{1}{4}\right)^4 [12+1] = \frac{13}{256}$ $\frac{1}{2}$

OR

Let E_1 : A coming on time.

E_2 : B coming on time.

$P(\bar{E}_1) = \frac{5}{7}, P(\bar{E}_2) = \frac{3}{7}$ $\frac{1}{2}$

P(only one on time)

$= P(E_1)\, P(\bar{E}_2) + P(\bar{E}_1)\, P(E_2)$

$= \frac{2}{7}\times\frac{3}{7}+\frac{5}{7}\times\frac{4}{7}$ 1

$= \frac{26}{49}$ $\frac{1}{2}$

7. $\begin{bmatrix} 2x-3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$ 1

$\Rightarrow 2x - 3 = 7$ and $2y - 4 = 14 \Rightarrow x = 5,\ y = 9 \Rightarrow x - y = -4$ 1

8. $(y+3x^2)\frac{dx}{dy} = x \ \Rightarrow\ \frac{dy}{dx} - \frac{1}{x}y = 3x$ $\frac{1}{2}$

I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log(x)} = \frac{1}{x}$ $\frac{1}{2}$

Solution is $y \cdot \frac{1}{x} = \int 3x \cdot \frac{1}{x} dx \Rightarrow y = 3x^2 + cx$ 1

9. Let $P(2, -1, 3), Q(3, -5, 1)$ and $R(-1, 11, 9)$ be three point.

$\overrightarrow{PQ} = \hat{i} - 4\hat{j} - 2\hat{k}$ $\frac{1}{2}$

$\overrightarrow{PR} = -3\hat{i} + 12\hat{j} + 6\hat{k} = -3(\hat{i} - 4\hat{j} - 2\hat{k})$ $\frac{1}{2}$

$\because$ $\overrightarrow{PR} = -3\overrightarrow{PQ}$, since P is common.

Therefore the points P, Q and R are collinear. 1

OR

$(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$

LHS = $(\vec{a} \times \vec{b})^2$

$= (|\vec{a}||\vec{b}| \sin\theta\, \hat{n})^2 = |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta$ 1

$= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2\theta)$

$= |\vec{a}|^2 |\vec{b}|^2 - (|\vec{a}||\vec{b}| \cos\theta)^2$

$= \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$ 1

10. $\int \frac{x-1}{(x-2)(x-3)} dx = \int \left(\frac{-1}{x-2} + \frac{2}{x-3} \right) dx$ 1

$= -\log|x-2| + 2\log|x-3| + C$ 1

OR

$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$ Put $e^x = t$ so that $e^x\, dx = dt$ $\frac{1}{2}$

$\int \frac{dt}{\sqrt{5 - 4t - t^2}} = \int \frac{dt}{\sqrt{3^2 - (t+2)^2}}$ 1

$= \sin^{-1}\left(\frac{t+2}{3}\right) + C = \sin^{-1}\left(\frac{e^x + 2}{3}\right) + C$ $\frac{1}{2}$

11. $\int e^x \left(\frac{2+\sin 2x}{2\cos^2 x}\right) dx = \int e^x (\sec^2 x + \tan x)\, dx$ 1

$= e^x \cdot \tan x + C$ 1

12. $P(A' \cap B') = P(A') \cdot P(B')$ 1

$= \frac{4}{7} \times \frac{3}{5}$

$= \frac{12}{35}$ 1

SECTION C

13. Equation of the required plane is

$a(x + 1) + b(y - 3) + c(z - 2) = 0$ ...(i) 1

Since plane (i) is perpendicular to planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$, therefore

$$\left.\begin{aligned} & a+2b+3c=0 \quad \text{...(ii)} \\ \text{and} \quad & 3a+3b+c=0 \quad \text{...(iii)} \end{aligned}\right\}$$ 1

Solving (ii) and (iii) we get

$\frac{a}{7} = \frac{b}{-8} = \frac{c}{3}$ 1

Equation of plane is

$7 \cdot (x + 1) - 8(y - 3) + 3(z - 2) = 0$

$\Rightarrow 7x - 8y + 3z + 25 = 0$ 1

14. $\int_1^5 (|x-1| + |x-2| + |x-4|\, dx$

$= \int_1^2 (5-x)dx + \int_2^4 (x+1)dx + \int_4^5 (3x-7)\, dx$ $1\frac{1}{2}$

$= \left[5x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} + x\right]_2^4 + \left[\frac{3x^2}{2} - 7x\right]_4^5$ $1\frac{1}{2}$

$= \frac{7}{2} + 8 + \frac{13}{2} = 18$ 1

15. $\Delta = 0$

$$\Rightarrow \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$ 1

$$\Rightarrow xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$ 1

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (xyz - 1) = 0$$ $\frac{1}{2}$

$$\Rightarrow \begin{vmatrix} 0 & x - z & x^2 - z^2 \\ 0 & y - z & y^2 - z^2 \\ 1 & z & z^2 \end{vmatrix} (xyz - 1) = 0$$ $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$ 1

$\Rightarrow (x - y)\ (y - z)\ (z - x)\ (xyz - 1) = 0$

$\because\ x \neq y \neq z$ therefore $xyz = 1$ $\frac{1}{2}$

16. LHS = $\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65}$

$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12} + \cos^{-1}\frac{63}{65}$ 1

$$= \tan^{-1}\left(\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}}\right) + \cos^{-1}\frac{63}{65}$$ 1

$= \tan^{-1}\left(\frac{63}{16}\right) + \cot^{-1}\left(\frac{63}{16}\right)$ 1

$= \frac{\pi}{2} = \text{RHS}$ 1

17. A(x, 5, –1), B(3, 2, 1), C(4, 5, 5), D(4, 2, –2)

$$\left.\begin{array}{l}\overrightarrow{BA} = (x-3)\hat{i} + 3\hat{j} - 2\hat{k} \\ \overrightarrow{BC} = 1\hat{i} + 3\hat{j} + 4\hat{k} \\ \overrightarrow{BD} = 1\hat{i} + 0\hat{j} - 3\hat{k}\end{array}\right\}$$ $1\frac{1}{2}$

$$\begin{vmatrix} x-3 & 3 & -2 \\ 1 & 3 & 4 \\ 1 & 0 & -3 \end{vmatrix} = 0$$ 1

i.e., $(x - 3)(-9) - 3(-7) - 2(-3) = 0$ 1

$\Rightarrow x = 6$ $\frac{1}{2}$

18. Let $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$, Put $x = \tan\theta$ $\frac{1}{2}$

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right) \Rightarrow y = \tan^{-1}(\tan 3\theta) = 3\theta$

$y = 3\tan^{-1} x \Rightarrow \frac{dy}{dx} = \frac{3}{1 + x^2}$ ...(i) 1

Let $z = \tan^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right)$, put $x = \sin\phi$ $\frac{1}{2}$

$z = \tan^{-1}\left(\frac{\sin\phi}{\sqrt{1 - \sin^2\phi}}\right) \Rightarrow z = \tan^{-1}(\tan\phi) = \phi$

$z = \phi = \sin^{-1} x \Rightarrow \frac{dz}{dx} = \frac{1}{\sqrt{1 - x^2}}$ ...(ii) 1

Using (i) & (ii), $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{3\sqrt{1 - x^2}}{1 + x^2}$ 1

OR

$\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, put $x = \sin\theta$, $y = \sin\phi$ 1

$\sqrt{1-\sin^2\theta}+\sqrt{1-\sin^2\phi}=a(\sin\theta-\sin\phi)$

$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow\ 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)=2a\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$ 1

$\Rightarrow\ \tan\left(\frac{\theta-\phi}{2}\right)=\frac{1}{a}$

$\Rightarrow\ \frac{\theta-\phi}{2}=\tan^{-1}\left(\frac{1}{a}\right)\Rightarrow \sin^{-1}x-\sin^{-1}y=2\tan^{-1}\left(\frac{1}{a}\right)$ 1

Differentiating both sides w.r.t x

$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$ $\frac{1}{2}$

$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$ or $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ $\frac{1}{2}$

19. (i) **Reflexive:** $\forall\ a \in A, |a-a|=0$ which is even

$\Rightarrow (a, a) \in R$, hence R is reflexive 1

(ii) **Symmetric:** Let $(a, b) \in R \Rightarrow |a-b|$ is even

$\Rightarrow |-(b-a)|$ is even $\Rightarrow |b-a|$ is even

so, $(b, a) \in R$

hence R is symmetric. 1

(iii) **Transitive:** Let $(a, b), (b, c) \in R$

so, $|a-b|$ is even and $|b-c|$ is even

$\Rightarrow a-b = 2\lambda,\ b-c = 2\mu$ where $\lambda, \mu \in Z$

Now, $a-c = (a-b)+(b-c) = 2(\lambda+\mu)$

$\Rightarrow |a-c|$ is even, so $(a, c) \in R$

hence R is transitive. $1\frac{1}{2}$

Since R is reflexive, symmetric and transitive therefore its an equivalence relation $\frac{1}{2}$

OR

Let for $x_1, x_2 \in A$, $f(x_1) = f(x_2)$ $\frac{1}{2}$

$$\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$$

$$\Rightarrow (4x_1+3)(6x_2-4) = (6x_1-4)(4x_2+3)$$

$\Rightarrow 34x_1 = 34x_2 \Rightarrow x_1 = x_2$, hence f is one-one. 1

For any $y \in A$ such that $y = \frac{4x+3}{6x-4}$ there exists x such that

$$6xy - 4y = 4x + 3 \Rightarrow (6y-4)x = 4y+3$$

$\Rightarrow x = \frac{4y+3}{6y-4}, y \in A, \; x = \frac{4y+3}{6y-4} \in A$ 1

$\Rightarrow$ f is onto. 1

Since f is one-one and onto, therefore f^{-1} exists in A

and $f^{-1}(y) = \frac{4y+3}{6y-4}$ or $f^{-1}(x) = \frac{4x+3}{6x-4}$ $\frac{1}{2}$

20. $(1+e^{2x})dy + (1+y^2)e^x \, dx = 0$

$\Rightarrow \int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx$ 1

$\Rightarrow \tan^{-1} y = -\int \frac{e^x}{1+e^{2x}} dx$ $\frac{1}{2}$

Put $e^x = t$, so that $e^x \, dx = dt$ $\frac{1}{2}$

$\tan^{-1} y = -\int \frac{dt}{1+t^2} \Rightarrow \tan^{-1} y = -\tan^{-1}(e^x) + C$ ...(i) 1

Substituting y = 1, when x = 0 in equation (i)

$\tan^{-1}(1) = -\tan^{-1}(1) + C \Rightarrow C = \frac{\pi}{2}$ $\frac{1}{2}$

Substituting $C = \frac{\pi}{2}$ in equation (i) $\Rightarrow \tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$ $\frac{1}{2}$

OR

$$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin\left(\frac{y}{x}\right)}$ ...(i) $\frac{1}{2}$

Put $\frac{y}{x} = v$ i.e., $y = vx$ in (i) so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$$v + x\frac{dv}{dx} = v - \frac{1}{\sin v}$$

$\int \sin v\, dv = \int -\frac{1}{x}\, dx$ $\frac{1}{2}$

$\Rightarrow -\cos v = -\log|x| + C \Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + C$ ...(ii) 1

Substituting $y = \frac{\pi}{2}$ when x = 1 in (ii)

$\cos\left(\frac{\pi}{2}\right) = \log 1 + C \Rightarrow C = 0$ $\frac{1}{2}$

Required solution is $\cos\left(\frac{y}{x}\right) = \log|x|$ $\frac{1}{2}$

21. $y = (\sin x)^x + \sin^{-1}\sqrt{1-x^2}$

Let $A = (\sin x)^x \Rightarrow \log A = x \log \sin x$ $\frac{1}{2}$

$\Rightarrow \frac{1}{A}\frac{dA}{dx} = \log \sin x + x \cot x$ 1

$\Rightarrow \frac{dA}{dx} = (\sin x)^x (\log \sin x + x \cot x)$ $\frac{1}{2}$

$B = \sin^{-1}\sqrt{1-x^2} = \cos^{-1} x$

$$\frac{dB}{dx} = \frac{-1}{\sqrt{1-x^2}}$$ 1

$$y = A + B \Rightarrow \frac{dy}{dx} = \frac{dA}{dx} + \frac{dA}{dx} = (\sin x)^x\ (\log \sin x + x \cot x) - \frac{1}{\sqrt{1-x^2}}$$ 1

22. $\int \cos 2x \cos 4x \cos 6x\, dx$

$= \frac{1}{2}\int 2\cos 6x \cos 2x \cdot \cos 4x\, dx$ $\frac{1}{2}$

$= \frac{1}{2}\int (\cos 8x + \cos 4x) \cos 4x\, dx$ 1

$= \frac{1}{4}\int (2\cos 8x \cos 4x + 2\cos^2 4x)\, dx$ $\frac{1}{2}$

$= \frac{1}{4}\int (\cos 12x + \cos 4x + 1 + \cos 8x)\, dx$ 1

$= \frac{1}{4}\left(\frac{\sin 12x}{12} + \frac{\sin 4x}{4} + x + \frac{\sin 8x}{8}\right) + C$ 1

23. $f(x) = \sin 2x + \cos 2x$

$f'(x) = 2 \cos 2x - 2 \sin 2x$ 1

$f'(x) = 0 \Rightarrow 2 \cos 2x - 2 \sin 2x = 0$

$\Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}$ 1

Possible interval are $\left(0, \frac{\pi}{8}\right), \left(\frac{\pi}{8}, \frac{5\pi}{8}\right), \left(\frac{5\pi}{8}, 2\pi\right)$ 1

$f'(x) < 0$ in $\left(\frac{\pi}{8}, \frac{5\pi}{8}\right) \Rightarrow f(x)$ is decreasing in $\left(\frac{\pi}{8}, \frac{5\pi}{8}\right)$ 1

SECTION D

24. Let $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Then A = IA

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \qquad (R_3 \to R_3 - 4R_2)$$ 4

$$\Rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -13 & 9 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \qquad (R_1 \to R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \qquad (R_1 \to R_2 - 3R_2)$$

$$\therefore\ A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$$ 1

OR

The given system of equations is

AX = B,

$$\text{where } A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$|A| = 1200 \neq 0$ 1

$\Rightarrow A^{-1}$ exists.

$X = A^{-1}B$ 1

$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ 2

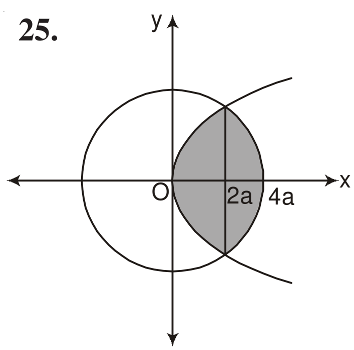
$$A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ $\frac{1}{2}$

$$X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$ 1

$\therefore\ x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$ $\frac{1}{2}$

25.



Correct figure 1

Solving $x^2 + y^2 = 16a^2$ ...(1)

and $y^2 = 6ax$ ...(2) we get

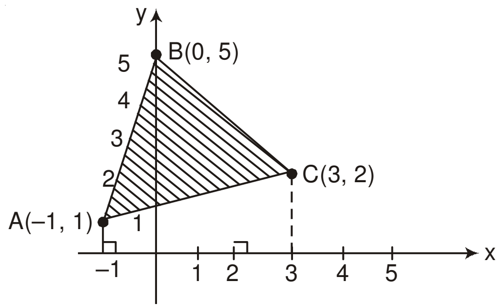
$x = 2a$ (as $-8a$ is not possible) 1

$$\text{Required Area} = 2\left[\int_0^{2a}\sqrt{6ax}\,dx + \int_{2a}^{4a}\sqrt{(4a)^2 - x^2}\,dx\right]$$ 2

$$= 2\left[\sqrt{6a}\left(\frac{2}{3}x^{3/2}\right)\right]_0^{2a} + 2\left[\frac{x}{2}\sqrt{(4a)^2 - x^2} + 8a^2\sin^{-1}\left(\frac{x}{4a}\right)\right]_{2a}^{4a}$$ 1

$$= \frac{8}{3}\sqrt{12}a^2 + \frac{16}{3}\pi a^2 - 4\sqrt{3}a^2$$

$$= \frac{4}{3}a^2(4\pi + \sqrt{3})$$ 1



OR

Correct figure 1

$4x - y + 5 = 0$ ...(1)

$x + y - 5 = 0$ ...(2)

$x - 4y + 5 = 0$ ...(3)

Coordinates of A(–1, 1), B(0, 5) and C(3, 2) $1\frac{1}{2}$

Required Area = $\int_{-1}^{0}(4x+5)dx + \int_{0}^{3}(5-x)dx - \frac{1}{4}\int_{-1}^{3}(x+5)dx$ 2

$= \left[2x^2+5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$ 1

$= \frac{15}{2}$ $\frac{1}{2}$

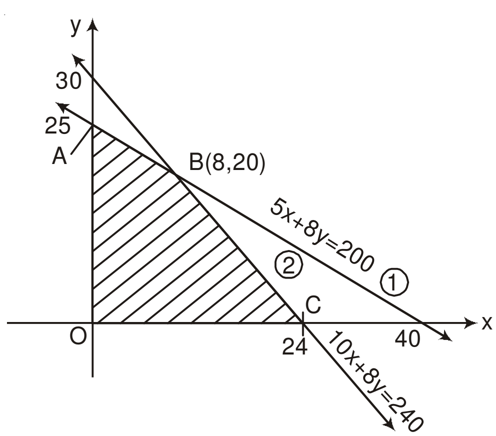
26. Let numbers of souvenirs of type A be x and number of souvenirs of type B be y

$\therefore$ L.P.P is

Maximize P = 100x + 120y $\frac{1}{2}$

Subject to constraints

$$\left.\begin{array}{ll} 5x+8y \le 200 & ...(1) \\ 10x+8y \le 240 & ...(2) \\ x, y \ge 0 & \end{array}\right\}$$ $2\frac{1}{2}$



2

Values at corner points

Points	P
A(0, 25)	3000
B(8, 20)	3200 (Max)
C(24, 0)	2400

1

So, 8 type A Souvenirs and 20 type B Souvenirs should be made to maximize profit.

27. Required equation of the line is

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + \hat{k})$ 2

Let $\vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}, \vec{a}_2 = \hat{i} + \hat{j}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

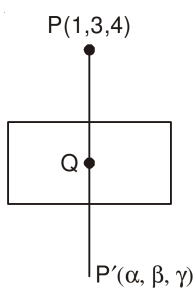
The required distance $= \left| \dfrac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$

$= \dfrac{|(\hat{i} - \hat{k}) \times (2\hat{i} - \hat{j} + \hat{k})|}{|2\hat{i} - \hat{j} + \hat{k}|}$ 1

$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix} = -\hat{i} - 3\hat{j} - \hat{k}$ 2

Required distance $= \dfrac{\sqrt{1+9+1}}{\sqrt{4+1+1}} = \dfrac{\sqrt{11}}{\sqrt{6}}$ or $\dfrac{\sqrt{66}}{6}$ 1

OR



Correct figure $\frac{1}{2}$

Equation of line PQ is $\dfrac{x-1}{2} = \dfrac{y-3}{-1} = \dfrac{z-4}{1} = \lambda$ 1

The coordinates of Q are $(2\lambda + 1, -\lambda + 3, \lambda + 4)$ $\frac{1}{2}$

$\because$ Q lies on plane $2x - y + z + 3 = 0$

$\therefore 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$ 1

$\Rightarrow 6\lambda + 6 = 0$ i.e., $\lambda = -1$ $\frac{1}{2}$

The coordinates of Q are (–1, 4, 3) $\frac{1}{2}$

$PQ = \sqrt{(-1-1)^2 + (4-3)^2 + (3-4)^2} = \sqrt{6}$ 1

Let $P'(\alpha, \beta, \gamma)$ be the image of P.

then $\frac{\alpha+1}{2} = -1, \ \frac{\beta+3}{2} = 4, \frac{\gamma+4}{2} = 3$ $\frac{1}{2}$

$\Rightarrow \alpha = -3, \ \beta = 5, \ \gamma = 2$

$\therefore$ the image P' is (–3, 5, 2) $\frac{1}{2}$

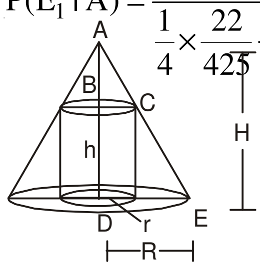
28. Let E_1 : spade card is lost
E_2 : non spade card is lost.
A : Two cards drawn are spade } 1

$P(E_1) = \frac{1}{4}, P(E_2) = \frac{3}{4}$ 1

$P(A \mid E_1) = \frac{^{12}C_2}{^{51}C_2} = \frac{22}{425}$ 1

$P(A \mid E_2) = \frac{^{13}C_2}{^{51}C_2} = \frac{26}{425}$ 1

$P(E_1 \mid A) = \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{3}{4} \times \frac{26}{425}} = \frac{11}{50}$ $1\frac{1}{2} + \frac{1}{2}$

29. 

Correct figure 1

$\Delta ABC \sim \Delta ADE$

$\frac{H-h}{H} = \frac{r}{R} \Rightarrow r = \frac{HR - hR}{H}$ 1

Now, $A = 2\pi h\left(\frac{HR - hR}{H}\right) = 2\pi hR - \frac{2\pi Rh^2}{H}$ 1

$$\frac{dA}{dh} = 2\pi R - \frac{4\pi Rh}{H}$$ 1

$$\frac{dA}{dh} = 0 \Rightarrow H = 2h$$ $\frac{1}{2}$

$$\left.\frac{d^2A}{dh^2}\right|_{H=2h} = \frac{-4\pi R}{2h} < 0 \Rightarrow \text{area is maximum}$$ 1

A is maximum at H = 2h.

$$\Rightarrow r = \frac{HR - \frac{HR}{2}}{H} = \frac{R}{2}$$ $\frac{1}{2}$