QUESTION PAPER CODE 30/5/3

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question numbers 1 to 10 are multiple choice questions of 1 mark each.

You have to select the correct choice :

Q.No.		Marks
1.	The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is **(a)** 4 **(b)** ±4 **(c)** –4 **(d)** 0 **Ans:** (b) ± 4	1
2.	Which of the following is *not* an A.P.? **(a)** –1.2, 0.8, 2.8, … **(b)** $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$ **(c)** $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$ **(d)** $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$ **Ans:** (c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$	1
3.	The radius of a sphere (in cm) whose volume is 12π cm^3, is **(a)** 3 **(b)** $3\sqrt{3}$ **(c)** $3^{2/3}$ **(d)** $3^{1/3}$ **Ans:** (c) $3^{2/3}$	1
4.	The distance between the points (m, –n) and (–m, n) is **(a)** $\sqrt{m^2+n^2}$ **(b)** m + n **(c)** $2\sqrt{m^2+n^2}$ **(d)** $\sqrt{2m^2+2n^2}$ **Ans:** (c) $2\sqrt{m^2+n^2}$	1
5.	In Figure 1, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If ∠QPR = 90°, then length of PQ is **(a)** 3 cm **(b)** 4 cm **(c)** 2 cm **(d)** $2\sqrt{2}$ cm **Ans:** (b) 4 cm	1
6.	On dividing a polynomial p(x) by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is **(a)** $3x^2 + x - 12$ **(b)** $x^3 - 4x + 3$ **(c)** $x^2 + 3x - 4$ **(d)** $x^3 - 4x - 3$ **Ans:** (b) $x^3 - 4x + 3$	1

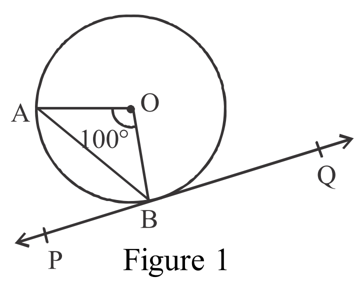
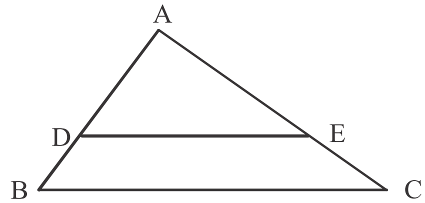
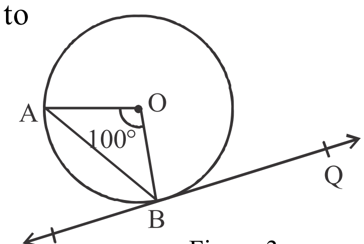


Figure 1

7.	In Figure 2, DE ∥ BC. If $\frac{AD}{DB} = \frac{3}{2}$ and AE = 2.7 cm, then EC is equal to **(a)** 2.0 cm **(b)** 1.8 cm **(c)** 4.0 cm **(d)** 2.7 cm  Figure 2 **Ans:** (b) 1.8 cm	1
8.	The point on the x-axis which is equidistant from (–4, 0) and (10, 0) is **(a)** (7, 0) **(b)** (5, 0) **(c)** (0, 0) **(d)** (3, 0) **Ans:** (d) (3, 0)	1
	OR The centre of a circle whose end points of a diameter are (–6,3) and (6, 4) is **(a)** (8, –1) **(b)** (4, 7) **(c)** $\left(0, \frac{7}{2}\right)$ **(d)** $\left(4, \frac{7}{2}\right)$ **Ans:** (c) $\left(0, \frac{7}{2}\right)$	1
9.	The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is **(a)** consistent **(b)** inconsistent **(c)** consistent with one solution **(d)** consistent with many solutions **Ans:** (b) inconsistent	1
10.	In Figure 3, PQ is tangent to the circle with centre at O, at the point B. If ∠AOB = 100°, then ∠ABP is equal to **(a)** 50° **(b)** 40° **(c)** 60° **(d)** 80°  Figure 3 **Ans:** (a) 50°	1
	Fill in the blanks in question numbers 11 to 15.	
11.	Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is __________. **Ans:** $\tan^2 A$	1
12.	If the probability of an event E happening is 0.023, then $P(\overline{E}) =$ __________. **Ans:** 0.977	1
13.	All concentric circles are __________ to each other. **Ans:** similar	1

14. The probability of an event that is sure to happen, is _________.

Ans: 1 — **1**

15. AOBC is a rectangle whose three vertices are A(0, –3), O(0, 0) and B(4, 0). The length of its diagonal is _________.

Ans: 5 units — **1**

Answer the following question numbers 16 to 20.

16. Write the value of $\sin^2 30° + \cos^2 60°$.

Ans: $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$ — **1/2**

$= \frac{1}{2}$ — **1/2**

17. Form a quadratic polynomial, the sum and product of whose zeros are (–3) and 2 respectively.

Ans: $x^2 + 3x + 2$ — **1**

OR

Can $(x^2 - 1)$ be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $(x^2 + 3)$? Justify your answer with reasons.

Ans: No, degree of remainder < degree of divisor — **1**

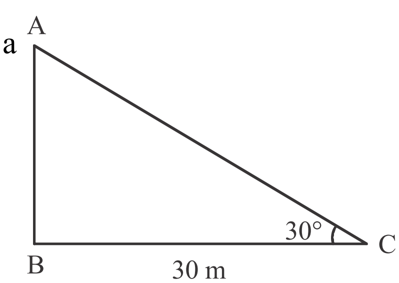
18. Find the sum of the first 100 natural numbers.

Ans: $\frac{100}{2}[2 + 99] = 5050$ — **1/2+1/2**

19. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.

Ans: $\frac{182 \times 13}{26} = 91$ — **1/2+1/2**

20. In Figure 4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of tower.



Ans: $\frac{AB}{30} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{30}{\sqrt{3}}$ m or $10\sqrt{3}$ m — **1/2+1/2**

SECTION – B

Question numbers 21 to 26 carry 2 marks each.

21. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes.

Ans: $\frac{\frac{1}{3}\pi r^2 (3h)}{\pi r^2 h}$ — **1**

$= \frac{1}{1} = 1 : 1$ — **1**

22. In Figure 5, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = BC + AD.

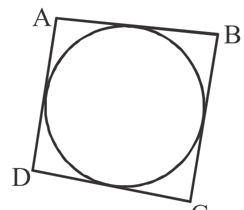
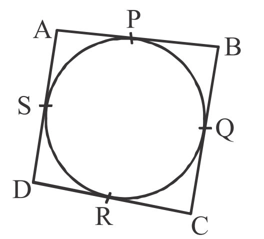


Figure 5

Ans: Let the circle touches the sides AB, BC, CD and AD at P, Q, R and S respectively.



1/2

$\therefore$ $\left.\begin{array}{l} AP = AS \\ BP = BQ \\ DR = DS \\ CR = CQ \end{array}\right\}$ 1

adding, we get (AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ) 1/2

$\therefore$ AB + CD = BC + AD

OR

In Figure 6, find the perimeter of ΔABC, if AP = 12 cm.

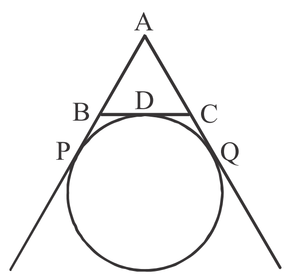


Figure 6

Ans: $\left.\begin{array}{l} AP = AB + BP = AB + BD \\ AQ = AC + CQ = AC + CD \end{array}\right\}$ 1

$\Rightarrow$ AP + AQ = AB + AC + (BD + CD) = AB + AC + BC

But AP = AQ $\therefore$ 2 AP = Perimeter of ABC 1/2

$\therefore$ Perimeter = 2(12) = 24 cm 1/2

23. Find the mode of the following distribution:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students:	4	6	7	12	5	6

Ans: Modal Group : 30 – 40 1/2

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 30 + \frac{5}{12} \times 10$$ 1

= 34.17 1/2

24. In the Figure 7, if PQ || BC and PR || CD, prove that $\frac{QB}{AQ} = \frac{DR}{AR}$.

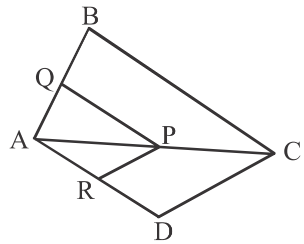


Figure 7

Ans: $\frac{QB}{AQ} = \frac{PC}{AP}$ ...(i) **1**

$\frac{PC}{AP} = \frac{DR}{AR}$ ...(ii) **1/2**

From (i) and (ii)

$\frac{QB}{AQ} = \frac{DR}{AR}$ **1/2**

25. Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

Ans: Let us assume that $5 + 2\sqrt{7}$ is not an irrational number.

$\therefore\ 5 + 2\sqrt{7}$ is a rational number p i.e. $5 + 2\sqrt{7} = p$ **1**

$\Rightarrow\ \sqrt{7} = \frac{p-5}{2}$ **1/2**

Which is a contradiction as RHS is a rational but LHS is irrational.

Hence $5 + 2\sqrt{7}$ can not be rational, so irrational. **1/2**

OR

Check whether 12^n can end with the digit 0 for any natural number n.

Ans: Prime factors of 12 are $2 \times 2 \times 3$ **1**

Since 5 is not a factor, so 12^n cannot end with 0. **1**

26. If A, B and C are interior angles of a ΔABC, then show that

$\cot\left(\frac{B+C}{2}\right) = \tan\left(\frac{A}{2}\right)$.

Ans: $A + B + C = 180°$, $\therefore\ \frac{B+C}{2} = 90° - \frac{A}{2}$ **1**

$\therefore\ \cot\left(\frac{B+C}{2}\right) = \cot\left(90° - \frac{A}{2}\right) = \tan\frac{A}{2}$ **1**

SECTION – C

Question numbers 27 to 34 carry 3 marks each.

27. Prove that:

$$\left(\sin^4\theta - \cos^4\theta + 1\right)\operatorname{cosec}^2\theta = 2$$

Ans: $\text{L.H.S} = \left[\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^2\theta - \cos^2\theta\right) + 1\right]\operatorname{cosec}^2\theta$ **1**

$= \left[1\left(\sin^2\theta - \cos^2\theta\right) - 1\right]\operatorname{cosec}^2\theta$ **1**

$= 2\sin^2\theta \times \operatorname{cosec}^2\theta = 2$ **1**

28. Find the sum:

$(-5) + (-8) + (-11) + \ldots + (-230)$

Ans: $a = -5,\ d = -3,\ a_n = -230$ **1/2**

$\Rightarrow \quad -5 + (n-1)\times(-3) = 230$ **1**

$(n-1) = \dfrac{225}{3} = 75$

$n = 76$ **1/2**

$S_{76} = \dfrac{76}{2}\left[-5 + (-230)\right]$ **1/2**

$= 38(-235) = -8930$ **1/2**

29. Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$.

Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

Ans: Constructing ΔABC with given dimensions **1**

Constructing the similar triangle. **2**

OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point.

Ans: Drawing a circle of radius 3.5 cm and centre O, and taking a point P such that OP = 7 cm **1**

Constructing two tangents. **2**

30. In Figure-8, ABCD is a parallelogram. A semicircle with centre O and the diameter AB has been drawn and it passes through D. If AB = 12 cm and OD ⊥ AB, then find the area of the shaded region. (Use $\pi = 3.14$)

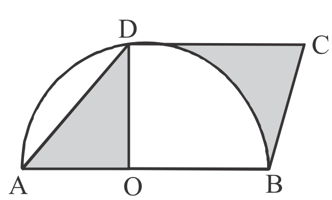


Figure 8

Ans: Area of shaded portion = Ar of ||gm – Ar of Quadrant — **1**

$$= 12 \times 6 - \frac{1}{4} \times 3.14 \times 6 \times 6$$ — **1**

$$= 43.75 \text{ cm}^2$$ — **1**

31. Read the following passage and answer the questions given at the end:

Diwali Fair

A game in a booth at Diwali fair involves using of spinner first. Then, if the spinner stops at an even number, the player is allowed to pick a marble from bag. The spinner and the marbles in the bag are represented in Figure-9

Prizes are given, when a black marble is picked. Shweta plays the game once.

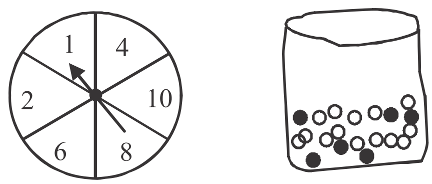


Figure 9

(i) What is the probability that she will be allowed to pick a marble from the bag?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?

Ans: (i) P(she will be allowed to pick a marble) = $\frac{5}{6}$ — **$1\frac{1}{2}$**

(ii) P(getting a prize) = $\frac{6}{20}$ or $\frac{3}{10}$ — **$1\frac{1}{2}$**

Both answers $\frac{6}{20}$ or $\frac{0}{20}$ for part (ii) in Q31 are to be treated correct as the bag contains marbles only.

32. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Ans: Let the fraction be $\frac{x}{y}$ — **1/2**

$$\therefore \frac{x-1}{y} = \frac{1}{3}, \frac{x}{y+8} = \frac{1}{4}$$ — **1/2+1/2**

$$\Rightarrow 3x - y = 3 , 4x - y = 8$$ — **1/2**

Solving to get x = 5, y = 12 $\therefore$ Fraction is $\frac{5}{12}$ — **1**

OR

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

Ans: Let the present age of son be x years

$\therefore$ Father's present age = (3x + 3) years. **1**

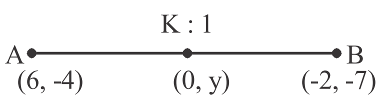
3 years hence, Son's age = (x + 3) years
and father's age = (3x + 6) years **1/2**

$\therefore \quad 3x + 6 = 2(x + 3) + 10$ **1**

$\Rightarrow \quad x = 10 \quad \therefore$ Son's age = 10 years,
Father's age = 33 years **1/2**

33. Find the ratio in which the y-axis divides the line segment joining the points (6, –4) and (–2, –7). Also find the point of intersection.

Ans:



Let the point P(0, y) on y-axis divides the line segment AB in K : 1 **1**

$$\therefore \quad 0 = \frac{-2K+6}{K+1} \Rightarrow K = 3 \quad \therefore \text{ Ratio is } 3 : 1$$ **1**

$$\text{Also, } y = \frac{3(-7)+1(-4)}{3+1} = \frac{-25}{4} \quad \therefore \text{ Point of intersection is } \left(0, \frac{-25}{4}\right)$$ **1**

OR

Show that the points (7, 10), (–2, 5) and (3, –4) are vertices of an isosceles right triangle.

Ans: Let the points be (7, 10), (–2, 5) and (3, –4)

$$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{106}$$ **1**

$$BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$$ **1/2**

$$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{212}$$ **1/2**

$AB = BC$ and $AC^2 = AB^2 + BC^2$

Hence ABC is isosceles right triangle. **1**

34. Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q + 1 for some integer q.

Ans: Any positive integer 'n' can be of the form 3m, 3m + 1, 3m + 2 **1**

$\therefore \quad n^2 = (3m)^2 = 9m^2 = 3(3m^2) = 3q,$

or $\quad n^2 = (3m+1)^2 = 9m^2 + 6m + 1 = 3(3m^2 + 2m) + 1 = 3q + 1,$

or $\quad n^2 = (3m+2)^2 = 9m^2 + 12m + 3 + 1$

$\qquad = 3(3m^2 + 4m + 1) + 1 = 3q + 1$ **$1\frac{1}{2}$**

Hence square of any positive integer is either of the form 3q or 3q + 1 for some integer q. **1/2**

SECTION – D

Question numbers 35 to 40 carry 4 marks each.

35. Sum of the areas of 2 squares is 544 m^2. If the difference of their perimeter is 32 m, find the sides of two squares.

Ans: Let 'a' and 'b' be the sides of two squares, with a > b.

then $a^2 + b^2 = 544$ and $4a - 4b = 32$ $1\frac{1}{2}$

or $a - b = 8 \quad \therefore \; a = b + 8$ 1

$\therefore \; (b + 8)^2 + b^2 = 544 \quad \Rightarrow \; 2b^2 + 16b - 480 = 0$ 1

$\therefore \; b^2 + 8b - 240 = 0 \quad \Rightarrow \; (b + 20)(b - 12) = 0 \quad \Rightarrow \; b = 12$

$b = 12$ m $\Rightarrow \; a = 12 + 8 = 20$ m 1/2

OR

A motorboat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans: Let speed of the stream be x km/h

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1$$ 2

$\Rightarrow \; 24(2x) = 324 - x^2$ or $x^2 + 48x - 324 = 0$ 1

$\Rightarrow \; (x + 54)(x - 6) = 0 \; \Rightarrow \; x = 6$ 1

$\therefore$ Speed of the stream = 6 km/h

36. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (In years):	0-10	10-20	20-30	0-40	40-50	50-60	60-70
Number of persons:	5	15	20	25	15	11	9

Ans: The points to be plotted for less than ogive are

(10, 5), (20, 20), (30, 40), (40, 65), (50 , 80), (60, 91), (70, 100) 2

Drawing the ogive $1\frac{1}{2}$

Getting median = 34 (approx) 1/2

OR

The distribution given below shows that the number of wickets taken by bowler in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets :	20-60	60-100	100-140	140-180	180-220	220-260
Number of bowlers :	7	5	16	12	2	3

No. of wickets :	20-60	60-100	100-140	140-180	180-220	220-260	Sum	
(f_i) No. of bowlers :	7	5	16	12	2	3	45	
x_i	40	80	120	160	200	240		
u_i	−2	−1	0	1	2	3		1/2
f_ix_i	−14	−5	0	12	4	9	6	1/2
cf	7	12	28	40	42	45		1/2

Mean = $a+\frac{\sum f_iu_i}{\sum f_i}\times h = 120+\frac{6\times 40}{45} = 125.33$ $1\frac{1}{2}$

Median = $l+\frac{\frac{N}{2}-c}{f}\times h = 100+\frac{22.5-12}{16}\times 40 = 126.25$ 1

37. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statute is 60° and from the same point the angle of elevation of the top of pedestal is 45°. Find the height of the pedestal. (Use $\sqrt{3}$ = 1.73)

Ans: For correct figure. 1

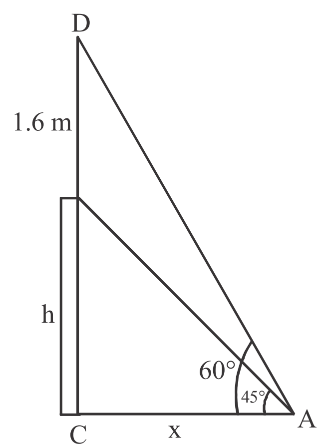
Let h m be the height of pedestal

Then from figure, $\frac{h}{x} = \tan 45° = 1$

and $\frac{h+1.6}{x} = \tan 60° = \sqrt{3}$ 1 + 1

$\Rightarrow \left(\sqrt{3}-1\right)h = 1.6$ 1/2

$\Rightarrow h = \frac{160}{73} = 2.19$ m (approx) 1/2



38. Obtain other zeroes of the polynomial

$$P(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$$

If two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Ans: Since $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of p(x), so $\left(x-\sqrt{5}\right)$ and $\left(x+\sqrt{5}\right)$ are factors of p(x). Thus $(x^2 - 5)$ is a factor of p(x). 1

$\left(2x^4 - x^3 - 11x^2 + 5x + 5\right) \div \left(x^2 - 5\right) = 2x^2 - x - 1$ $1\frac{1}{2}$

$2x^2 - x - 1 = (2x + 1)(x - 1)$ 1

$\therefore$ Other zeroes of p(x) are $1, -\frac{1}{2}$ 1/2

	OR What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$? **Ans:** $$\begin{array}{r l} & 2x+5 \\ x^2-4x+8 \,\big) & 2x^3-3x^2+6x+7 \\ - & \underline{2x^3 \mathop{-}\limits_{+} 8x^2 \mathop{+}\limits_{-} 16x} \\ & 5x^2-10x+7 \\ - & \underline{5x^2 \mathop{-}\limits_{+} 20x \mathop{+}\limits_{-} 40} \\ & 10x-33 \end{array}$$	3
	$\therefore$ We have to add $(33 - 10x)$	1
39.	In a cylindrical vessel of radius 10 cm, containing some water, 9000 small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0.5 cm, then find the rise in the level of water in the vessel.	
	Ans: Volume of raised water in cylinder = Volume of 9000 spherical balls	1
	$\pi(10)^2 H = 9000 \times \frac{4}{3} \times \pi \times (0.5)^3$	2
	$\therefore$ H = 15 cm	1
40.	If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.	
	Ans: For correct Given, To Prove, Constructions and figure	$\frac{1}{2} \times 4 = 2$
	For correct proof	2