Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/2/2

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/2/2

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No.		Marks
1.	The area of a triangle formed by vertices O, A and B, where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}\ \hat{j}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is **(A)** $3\sqrt{5}$ sq. units **(B)** $5\sqrt{5}$ sq. units **(C)** $6\sqrt{5}$ sq. units **(D)** 4 sq. units **Ans:** (A) $3\sqrt{5}$ sq. units	1
2.	If $\cos\left(\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x\right) = 0$, then x is equal to **(A)** $\frac{1}{\sqrt{5}}$ **(B)** $-\frac{2}{\sqrt{5}}$ **(C)** $\frac{2}{\sqrt{5}}$ **(D)** 1 **Ans:** (C) $\frac{2}{\sqrt{5}}$	1
3.	The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is **(A)** $(-\infty, \infty)$ **(B)** $(-\infty, 0)$ **(C)** $(2, \infty)$ **(D)** $(0, 2)$ **Ans:** (D) (0, 2)	1
4.	The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at **(A)** exactly one point **(B)** exactly two point **(C)** exactly three point **(D)** no point **Ans:** Solution not provided	1
	(One mark to be given to all)	
5.	The function f : R → [–1, 1] defined by f(x) = cos x is **(A)** both one-one and onto **(B)** not one-one, but onto **(C)** one-one, but not onto **(D)** neither one-one, nor onto **Ans:** (B) not one-one, but onto	1

6. The coordinates of the foot of the perpendicular drawn from the point (2, –3, 4) on the y-axis is

(A) (2, 3, 4) **(B)** (–2, –3, –4)

(C) (0, –3, 0) **(D)** (2, 0, 4)

Ans: (C) (0, –3, 0) — 1

7. The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is

(A) symmetric and transitive, but not reflexive

(B) reflexive and symmetric, but not transitive

(C) symmetric, but neither reflexive nor transitive

(D) an equivalence relation

Ans: (C) symmetric, but neither reflexive nor transitive — 1

8. The angle between the vectors $\hat{i}-\hat{j}$ and $\hat{j}-\hat{k}$ is

(A) $-\frac{\pi}{3}$ **(B)** 0 **(C)** $\frac{\pi}{3}$ **(D)** $\frac{2\pi}{3}$

Ans: (D) $\frac{2\pi}{3}$ — 1

9. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of |A| is

(A) –3 **(B)** 3 **(C)** 9 **(D)** 27

Ans: (D) 27 — 1

10. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then $|\lambda\vec{a}|$ lies in

(A) [0, 12] **(B)** [2, 3] **(C)** [8, 12] **(D)** [–12, 8]

Ans: (A) [0, 12] — 1

Fill in the blanks in questions numbers 11 to 15

11. If the radius of the circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is ________

Ans: π cm/s — 1

12. If $\begin{vmatrix} 2x & -9 \\ -2 & x \end{vmatrix} = \begin{vmatrix} -4 & 8 \\ 1 & -2 \end{vmatrix}$, the value of x is ____________.

Ans: 3 or –3 — 1

13. The corner points of the feasible region of an LPP are (0,0), (0,8), (2,7), (5,4) and (6,0). The maximum profit $P = 3x + 2y$ occurs at the point ________

Ans: (5, 4) **1**

14. The range of the principal value branch of the function $y = \sec^{-1} x$ is __________.

Ans: $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ **1**

OR

The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is ____________.

Ans: $\frac{2\pi}{3}$ **1**

15. The distance between parallel planes $2x + y - 2z - 6 = 0$ and $4x + 2y - 4z = 0$ ______ units.

Ans: 2 **1**

OR

If P(1,0,–3) is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is ________

Ans: $x - 3z = 10$ **1**

Question numbers 16 to 20 are very short answer type questions

16. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx$.

Ans: Let $f(x) = x \cos^2 x$ $\quad f(-x) = -f(x)$ or f is odd **1/2**

$\therefore \int_{-\pi/2}^{\pi/2} x \cos^2 x dx = 0$ **1/2**

17. Find the coordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the line xy-plane.

Ans: Putting z = 0 in given equation gives $\frac{x-1}{3} = \frac{y+4}{7} = 2$ **1/2**

Coordinates of required point are (7, 10, 0) **1/2**

18. Find the value of k, so that the function $f(x) = \begin{cases} kx^2 + 5, & \text{if} \quad x \le 1 \\ 2 \quad , & \text{if} \quad x > 1 \end{cases}$

is continuous at x = 1.

Ans: L.H.L. is k + 5 — 1/2

getting k = –3 — 1/2

19. Find the integrating factor of the differential equation

$$x\frac{dy}{dx} = 2x^2 + y$$

Ans: Integrating factor is $e^{\int \frac{-1}{x}dx}$ or $\left\{ \begin{array}{l} \text{writing given equation as} \\ \frac{dy}{dx} - \frac{y}{x} = 2x \end{array} \right.$ — 1/2

$= \frac{1}{x}$ — 1/2

20. Differentiate $\sec^2(x^2)$ with respect to x^2.

Ans: Let $x^2 = u$, differentiating $\sec^2 u$ w.r.t u, putting $= x^2$ — 1/2

$2\sec^2 x^2 \tan x^2$ — 1/2

OR

If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$.

Ans: $\frac{dy}{dx} = f'(x^2)2x$ — 1/2

$= 2xe^x$ — 1/2

SECTION-B

Question numbers 21 to 26 carry 2 marks.

21. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

Ans: Let the vector $\vec{r} = a\hat{i} + a\hat{j} + a\hat{k}$ — 1/2

$\therefore \quad \sqrt{3a^2} = 3\sqrt{3}$ — 1

required vector is $3\hat{i} + 3\hat{j} + 3\hat{k}$ or $-3\hat{i} - 3\hat{j} - 3\hat{k}$ — 1/2

OR

Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\,\vec{a} - \vec{b}$ is also a unit vector.

Ans: Using $\left|\sqrt{3}\,\vec{a}-\vec{b}\right|=1$ i.e. $\left|\sqrt{3}\,\vec{a}-\vec{b}\right|^2=1$ **1/2**

& getting $\vec{a}\cdot\vec{b}=\dfrac{\sqrt{3}}{2}$ **1**

getting angle $\dfrac{\pi}{6}$ or $30°$ **1/2**

22. If $A=\begin{bmatrix}-3 & 2\\ 1 & -1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find scalar k so that $A^2+I=kA$.

Ans: $A^2 = \begin{bmatrix}11 & -8\\ -4 & 3\end{bmatrix}$ **1**

$A^2+I=kA \Rightarrow \begin{bmatrix}12 & -8\\ -4 & 4\end{bmatrix}=\begin{bmatrix}-3k & 2k\\ k & -k\end{bmatrix}$ **1/2**

$k = -4$ **1/2**

23. If $f(x)=\sqrt{\dfrac{\sec x-1}{\sec x+1}}$, find $f'\left(\dfrac{\pi}{3}\right)$.

Ans: $f(x) = \sqrt{\dfrac{\dfrac{1}{\cos x}-1}{\dfrac{1}{\cos x}+1}} = \tan\dfrac{x}{2}$ **1**

$f'(x) = \dfrac{1}{2}\sec^2\dfrac{x}{2}$ **1/2**

$f'\left(\dfrac{\pi}{3}\right) = \dfrac{2}{3}$ **1/2**

OR

Find $f'(x)$ if $f(x)=(\tan x)^{\tan x}$.

Ans: Taking log on both sides. $\log f(x) = \tan x \log \tan x$ **1/2**

differentiating to get $\dfrac{f'(x)}{f(x)}=\sec^2 x+\sec^2 x\log\tan x$ **1**

Thus, $f'(x) = (\tan x)^{\tan x}\cdot\sec^2 x(1+\log\tan x)$ **1/2**

24. Find $\int\dfrac{\tan^3 x}{\cos^3 x}dx$.

Ans: Given Integral is $I = \int\dfrac{\sin^3 x}{\cos^6 x}dx$ **1/2**

Put $\cos x = t$

$\sin x\, dx = -dt$ **1/2**

$$= \int\left[\frac{-1}{t^6}+\frac{1}{t^4}\right]dt$$

$$= \frac{t^{-5}}{5}-\frac{t^{-3}}{3}+c$$ 1/2

$$= \frac{1}{5(\cos x)^5}-\frac{1}{3(\cos x)^3}+c \text{ or } \frac{\sec^5 x}{5}-\frac{\sec^3 x}{3}+c$$ 1/2

25. Show that the plane x – 5y – 2z = 1 contains the line $\frac{x-5}{3}=y=2-z$

Ans: Given point on line 5, 0, 2

Putting in equation of plane 5 – 0 – 4 = 1

1 = 1 ∴ point lies on plane 1

dr's of line 3, 1, –1

dr's of normal to the plane 1, –5, –2

As 3 · 1 + 1 · –5 + –1 · –2 = 0 1

∴ given plane contains given line.

26. A fair dice is thrown two times. find the probability distribution of the number of sixes. Also determine the mean of the number of sixes.

Ans: X denote number of sixes $P(6)=\frac{1}{6}, P(\overline{6})=\frac{5}{6}$ 1/2

X	0	1	2		
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$	...	1
XP(X)	0	$\frac{10}{36}$	$\frac{2}{36}$		

mean $= \sum XP(X)=\frac{1}{3}$ 1/2

SECTION-C

Question numbers 27 to 32 carry 4 marks.

27. Solve the following differential equation:

$$\left(1+e^{y/x}\right)dy+e^{y/x}\left(1-\frac{y}{x}\right)dx=0 \ (x\neq 0)$$

Ans: Writing given differential equation as $\frac{dy}{dx}=\frac{e^{y/x}(y/x-1)}{1+e^{y/x}}$ 1

Putting $y = vx$ & $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\frac{1}{2}$

to get $x\frac{dv}{dx} = \frac{-(e^v + v)}{1 + e^v}$ **1**

$$\Rightarrow \int \frac{e^v + 1}{e^v + v} dv = -\int \frac{dx}{x}$$

$\log|e^v + v| = -\log|x| + \log c$ **1**

$e^{y/x} + \frac{y}{x} = \frac{c}{x}$ or $x e^{y/x} + y = c$ **$\frac{1}{2}$**

28. A cotton industry manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table

Items	Machine time	Craftsman time	Profit (in ₹)
Pedestal lamp	1.5 hours	3 hours	30
Wooden shades	3 hours	1 hour	20

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacture schedule his daily production in order to maximize the profit? Formulate it as an LPP and solve it graphically.

Ans: Let number of pedestal lamps = x
number of wooden shades = y

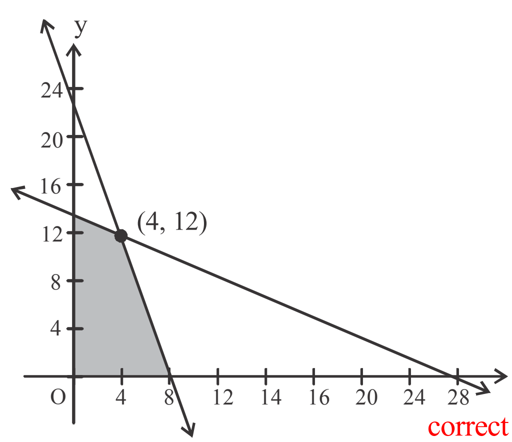
Maximize Profit Z = 30x + 20y **$\frac{1}{2}$**

Subject to constraints

$1.5x + 3y \leq 42$

$3x + y \leq 24$ **$1\frac{1}{2}$**

$x \geq 0, y \geq 0$



correct graph **1**

getting corners points & values of Z

(0, 0)	0
(8, 0)	240
(4, 12)	360
(0, 14)	280

1/2

Maximum profit = ₹ 360 where x = 4, y = 12 **1/2**

29. Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx$

Ans: Writing $I = \int_0^{\frac{\pi}{2}} 2\sin x \cos x \tan^{-1}(\sin x)dx$ **1/2**

putting sinx = t
cosxdx = dt **1/2**

$\Rightarrow I = \int_0^1 2t \tan^{-1}t\, dt$ **1/2**

$= \left. t^2 \tan^{-1} t - \int \frac{t^2}{1+t^2} dt \right]_0^1$ **1**

$= \left. t^2 \tan^{-1} t - t + \tan^{-1} t \right]_0^1$ **1**

$= \frac{\pi}{2} - 1$ **1/2**

30. Check whether the relation R in the set N of natural numbers given by

$R = \{(a,b): a \text{ is divisor of } b\}$

is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

Ans: For reflexive

Let $a \in N$ clearly a divides a $\therefore$ $(a, a) \in R$

$\therefore$ R is reflexive **1**

For symmetric

$(1, 2) \in R$ but $(2, 1) \notin R$ **1**

$\therefore$ R is not symmetric

For transitive

Let $(a, b), (b, c) \in R$

$\therefore$ a divides b and b divides c

$\Rightarrow$ a divides c $\therefore$ $(a, c) \in R$ **1**

R is transitive

As R is not symmetric $\therefore$ It is not an equivalence relation **1**

OR

Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$.

Ans: $\text{LHS} = \tan^{-1}\left(\dfrac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right) = \tan^{-1}\frac{1}{2}$ **2**

$= \frac{1}{2}\cdot 2\tan^{-1}\frac{1}{2}$ **1**

$= \frac{1}{2}\sin^{-1}\left(\dfrac{2\times\frac{1}{2}}{1+\frac{1}{4}}\right) = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$ **1**

$=$ RHS

31. Find the equation of the plane passing through the points (1, 0, –2), (3, –1, 0) and perpendicular to the plane 2x – y + z = 8. Also find the distance of the plane thus obtained from the origin.

Ans: Let P(1, 0, –2) and Q(3, –1, 0)

dr's of line through PQ 2, –1, 2 **1/2**

Let dr's of Normal of required plane be A, B, C

$\therefore \left.\begin{array}{l} 2A - B + 2C = 0 \\ \& \; 2A - B + C = 0 \end{array}\right\}$ **1**

Solving we get A = 1, B = 2, C = 0 **1**

Equation of plane is 1(x – 1) + 2 (y – 0) + 0(z + 2) = 0

i.e. x + 2y = 1 **1/2**

Distance of above plane from origin is $\left|\dfrac{0+0+0-1}{\sqrt{1+4}}\right|$

$= \dfrac{1}{\sqrt{5}} \text{ or } \dfrac{\sqrt{5}}{5}$ **1**

32. If $\tan^{-1}\left(\frac{y}{x}\right)=\log\sqrt{x^2+y^2}$, prove that $\frac{dy}{dx}=\frac{x+y}{x-y}$.

Ans: Differentiating both sides w.r.t. x to get

$$\frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{x\frac{dy}{dx}-y}{x^2} = \frac{1}{\sqrt{x^2+y^2}}\cdot\frac{2x+2y\frac{dy}{dx}}{2\sqrt{x^2+y^2}}$$ **$1\frac{1}{2}+1\frac{1}{2}$**

Simplyfying we get $x\frac{dy}{dx} - y = x + y\frac{dy}{dx}$ $\frac{1}{2}$

getting $\frac{dy}{dx} = \frac{x+y}{x-y}$ $\frac{1}{2}$

OR

If $y = e^{a\cos^{-1}x}, -1 < x < 1$, then show that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

Ans: $\frac{dy}{dx} = \frac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$ 1

$\sqrt{1-x^2}\frac{dy}{dx} = -ae^{a\cos^{-1}x}$ $\frac{1}{2}$

Differentiating again & getting

$\sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\cdot\frac{dy}{dx} = \frac{a^2e^{a\cos^{-1}x}}{\sqrt{1-x^2}}$ 2

$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$ $\frac{1}{2}$

SECTION-D

Question numbers 33 to 36 carry 6 marks.

33. Amongst all open (from the top) right circular cylindrical boxes of volume 125π cm^3, find the dimensions of the box which has the least surface area.

Ans: Let radius = r & height = h

$\pi r^2 h = 125\pi$ or $h = \frac{125}{r^2}$ 1

Surface Area, $S = 2\pi r h + \pi r^2$

$S = \frac{250\pi}{r} + \pi r^2$ 1

$\frac{dS}{dr} = \frac{-250\pi}{r^2} + 2\pi r$ 1

$\frac{dS}{dr} = 0 \Rightarrow r = 5$ 1/2+1

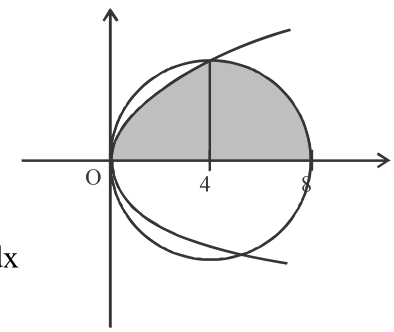
$\frac{d^2S}{dr^2} = \frac{500\pi}{r^3} + 2\pi > 0$ so s is least 1

when r = 5 cm , gives h = 5 cm 1/2

34. Using integration, find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

Ans. Correct figure **1**

x-coordinate of point of intersection is 4, 0 **1**



Required Area $= \int_0^4 2\sqrt{x}\, dx + \int_4^8 \sqrt{16-(x-4)^2}\, dx$ **2**

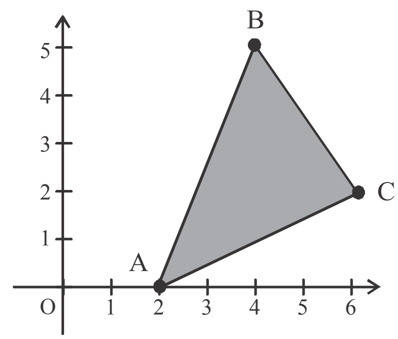
$$= \frac{4}{3}x^{\frac{3}{2}}\Big|_0^4 + \frac{x-4}{2}\sqrt{16-(x-4)^2} + 8\sin^{-1}\frac{x-4}{4}\Big|_4^8$$ **1**

$$= \frac{32}{3} + 4\pi$$ **1**

OR

Using the method of integration, find the area of the ΔABC, coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3).

Ans. Correct figure **1**



Equation of AB : $y = \frac{5}{2}(x-2)$ **1/2**

Equation of BC : $y = 9 - x$ **1/2**

Equation of AC : $y = \frac{3}{4}(x-2)$ **1/2**

Required Area $= \frac{5}{2}\int_2^4 (x-2)\, dx + \int_4^6 (9-x)\, dx - \frac{3}{4}\int_2^6 (x-2)\, dx$ **2**

$$= \frac{5}{2}\frac{(x-2)^2}{2}\Big|_2^4 + \frac{(9-x)^2}{-2}\Big|_4^6 - \frac{3}{4}\frac{(x-2)^2}{2}\Big|_2^6$$ **1**

$= 7$ **1/2**

35. $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

Ans: $|A| = 51$ **1**

Cofactors:
$$\begin{matrix} A_{11} = 28 & A_{12} = 13 & A_{13} = -19 \\ A_{21} = -2 & A_{22} = 10 & A_{23} = 5 \\ A_{31} = -17 & A_{32} = -17 & A_{33} = 17 \end{matrix}$$ **2**

$$A^{-1} = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$ **1**

Given system is AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$ $\frac{1}{2}$

$$\Rightarrow X = A^{-1}B = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$ **1**

x = 3, y = 2, z = −2 $\frac{1}{2}$

OR

If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then using properties of determinants show that 1 + xyz = 0.

Ans: Writing L.H.S as $\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$ **1**

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$ **1**

getting

$$(1+xyz)\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$ **2**

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$(1+xyz)\begin{vmatrix} x & x^2 & 1 \\ y-x & (y+x)(y-x) & 0 \\ z-x & (z+x)(z-x) & 0 \end{vmatrix} = 0$$ **1**

Expanding and simplifying to get

$\Rightarrow (1 + xyz)\ (x - y)\ (y - z)\ (z - x) = 0$ **1/2**

As x, y, z are different **1/2**

$\therefore\ 1 + xyz = 0$

36. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn randomly one-by-one without replacement and are found to be both kings. Find the probability of the lost card being a king.

Ans: Let E_1 : Lost card is king

Let E_2 : Lost card is not a king **1/2**

A : Two cards drawn are kings

$$P(E_1) = \frac{1}{13},\ P(E_2) = \frac{12}{13}$$ **1**

$$P(A/E_1) = \frac{{}^3C_2}{{}^{51}C_2},\ P(A/E_2) = \frac{{}^4C_2}{{}^{51}C_2}$$ **1+1**

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{13} \times \frac{{}^3C_2}{{}^{51}C_2}}{\frac{1}{13} \times \frac{{}^3C_2}{{}^{51}C_2} + \frac{12}{13} \times \frac{{}^4C_2}{{}^{51}C_2}}$$ **$1\frac{1}{2}$**

$$= \frac{3}{75} \text{ or } \frac{1}{25}$$ **1**