QUESTION PAPER CODE 65/2/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} = 2ae^{2x} \Rightarrow \frac{dy}{dx} = 2(y-5)$ $\frac{1}{2}+\frac{1}{2}$

2. $\frac{dy}{dx} = -\frac{\sqrt{3}\sin(\sqrt{3x})}{2\sqrt{x}}$ 1

3. $|A'|\,|A| = |I| \Rightarrow |A|^2 = 1$ $\frac{1}{2}$

$\Rightarrow |A| = 1$ or $|A| = -1$ $\frac{1}{2}$

4. D. Rs are 1, 1, 1
$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ 1

SECTION B

5. $\overrightarrow{AB} = 3\hat{i} - \hat{j} - 2\hat{k};\ \overrightarrow{AC} = 9\hat{i} - 3\hat{j} - 6\hat{k}$ 1

Clearly, $\overrightarrow{AC} = 3\cdot\overrightarrow{AB} \Rightarrow \overrightarrow{AC} \parallel \overrightarrow{AB}$, $\therefore$ A, B & C are Collinear 1

OR

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = -17\hat{i} + 13\hat{j} + 7\hat{k}$ $1\frac{1}{2}$

$\therefore\ |\vec{a}\times\vec{b}| = \sqrt{289+169+49} = \sqrt{507}$ $\frac{1}{2}$

6. $\int \frac{x-5}{(x-3)^3} \cdot e^x \, dx = \int e^x \left[\frac{(x-3)-2}{(x-3)^3} \right] dx$ $\frac{1}{2}$

$= \int e^x [(x-3)^{-2} - 2(x-3)^{-3}] dx$ $\frac{1}{2}$

$= e^x (x-3)^{-2} + c$ or $\frac{e^x}{(x-3)^2} + c$ 1

7. $\left.\begin{array}{l} \forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R} \end{array}\right\}$ 1

Also,

$\left.\begin{array}{l} a*(b*c) = a*\sqrt{b^2+c^2} = \sqrt{a^2+b^2+c^2} \\ (a*b)*c = \sqrt{a^2+b^2}*c = \sqrt{a^2+b^2+c^2} \end{array}\right\} \left.\begin{array}{l} \Rightarrow\ a*(b*c) = (a*b)*c \\ \therefore\ * \text{ is Associative} \end{array}\right\}$ 1

8. $(A - 2I)\ (A - 3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ 1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ 1

9. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \cdot \tan x + \operatorname{cosec} x \cdot \cot x)\, dx$ 1

$= \sec x - \operatorname{cosec} x + c$ 1

OR

$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{(x-1)^{-2} - 2(x-1)^{-3}\} dx$ 1

$\left.\begin{array}{c} = e^x (x-1)^{-2} + c \\ \text{or} \\ \frac{e^x}{(x-1)^2} + c \end{array}\right\}$ 1

10. (i) $P(3 \text{ heads}) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{5}{16}$ 1

(ii) P(At most 3 heads) = P $(r \leq 3)$

$$= 1 - P(4 \text{ heads or } 5 \text{ heads})$$
$$= 1 - \left\{ {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5 \left(\frac{1}{2}\right)^5 \right\}$$
$$= \frac{26}{32} \text{ or } \frac{13}{16}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2	$\frac{1}{2}$
P(x):	1/4	1/2	1/4	$1\frac{1}{2}$

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B/A) = 0.5 \times 0.3 = 0.15$ $\frac{1}{2}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70} \text{ or } \frac{3}{14}$ 1

12. Differenatiae $y = Ae^{2x} + Be^{-2x}$, we get

$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$, differenatiate again to get, 1

$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y \text{ or } \frac{d^2y}{dx^2} - 4y = 0$ 1

SECTION C

13. $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

$\Rightarrow \tan^{-1}\left\{\frac{2x}{1-(x^2-1)}\right\} = \tan^{-1}\frac{8}{31}$ 2

$\Rightarrow 4x^2 + 31x - 8 = 0 \Rightarrow x = \frac{1}{4} \text{ or } x = -8$ 1

x = –8 does not satisty the given equation so $x = \frac{1}{4}$ 1

14. $x = ae^t(\sin t + \cos t);\ y = ae^t(\sin t - \cos t)$

then

$$\left.\begin{aligned}\frac{dy}{dt} &= ae^t(\sin t - \cos t) + ae^t(\cos t + \sin t)\\ &= y + x\end{aligned}\right\}$$ $1\frac{1}{2}$

$$\left.\begin{aligned}\frac{dx}{dt} &= ae^t(\sin t + \cos t) + ae^t(\cos t - \sin t)\\ &= x - y\end{aligned}\right\}$$ $1\frac{1}{2}$

$\therefore\ \frac{dy}{dx} = \frac{y+x}{x-y}$ or $\frac{x+y}{x-y}$ 1

OR

Let, $y = x^{\sin x} + (\sin x)^{\cos x} = u + v;\ \therefore\ \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 1

$u = x^{\sin x} \Rightarrow \log u = \sin x \cdot \log x \Rightarrow \frac{du}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\}$ $1\frac{1}{2}$

$v = (\sin x)^{\cos x} \Rightarrow \log v = \cos x \cdot \log(\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x \cdot \log \sin x\}$ 1

$\therefore\ \frac{dy}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x . \log \sin x\}$ $\frac{1}{2}$

15. $\int \frac{2\cos x}{(1-\sin x)(2-\cos^2 x)}dx = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx = \int \frac{2}{(1-t)(1+t^2)}dt$, where $\sin x = t$, $\cos x\, dx = dt$

1

$\int \frac{2}{(1-t)(1+t^2)}dt = \int \frac{1}{1-t}dt + \int \frac{t+1}{t^2+1}dt$ $1\frac{1}{2}$

$= -\log(1-t) + \frac{1}{2}\log(t^2+1) + \tan^{-1}(t) + c$ 1

$= -\log|1-\sin x| + \frac{1}{2}\log(\sin^2 x + 1) + \tan^{-1}(\sin x) + c$ $\frac{1}{2}$

16. $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A$, $(1, 1) \notin R \Rightarrow$ R is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow$ R is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow$ R is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

$\therefore$ 'f' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \rightarrow N$ and $f^{-1}(y) = \frac{y-3}{4}$ or $f^{-1}(x) = \frac{x-3}{4}, x \in Y$ 1

17. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \rightarrow C_1 + C_2 + C_3 \end{pmatrix}$$ 1

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \left\{ \begin{array}{l} \text{By applying,} \\ R_2 \rightarrow R_2 - R_1; \\ R_3 \rightarrow R_3 - R_1 \end{array} \right.$$ 2

$= (a + b + c) \{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)(ab + bc + ca)$ 1

18. $(x - a)^2 + (y - b)^2 = c^2, c > 0$

Differentiating both sides with respect to 'x', we get

$$2(x - a) + 2(y - b) \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x-a}{y-b}$$ $1\frac{1}{2}$

Differentiating again with respect to 'x', we get;

$$\frac{d^2y}{dx^2} = -\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2} = \frac{-c^2}{(y-b)^3} \quad \text{(By substituting '}\frac{dy}{dx}\text{')}$$ $1+\frac{1}{2}$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c$$ 1

Which is a constant independent of 'a' & 'b'.

19. Let (α, β) be the point on the curve where normal

passes through $(-1, 4)$ $\therefore$ $\alpha^2 = 4\beta$, also $\frac{dy}{dx} = \frac{x}{2}$

Slope of normal at (α, β) = $\frac{-1}{\left.\frac{dy}{dx}\right]_{(\alpha,\beta)}} = \frac{-1}{\frac{\alpha}{2}} = \frac{-2}{\alpha}$ 1

Equation of normal: $y-4 = \frac{-2}{\alpha}(x+1)$ $\frac{1}{2}$

(α, β) lies on normal $\Rightarrow \beta - 4 = \frac{-2}{\alpha}(\alpha+1)$

Putting $\beta = \frac{\alpha^2}{4}$, we get; $\alpha^3 - 8\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha^2 + 2\alpha - 4) = 0$ 1

For $\alpha = 2$ Equation of normal is: $x + y - 3 = 0$ 1

For $\alpha = \pm\sqrt{5}-1$; Equation of normal is: $y-4 = \frac{-2}{\pm\sqrt{5}-1}(x+1)$ $\frac{1}{2}$

20. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)dx$ Put $x = a - t,\ dx = -dt$

Upper limit = $t = a - x = a - a = 0$

Lower limit = $t = a - x = a - 0 = a$ 1

$$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx$$

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$\therefore\ I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x} dx$ ...(ii) 1

Adding (i) and (ii) we get

$$2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx = \frac{\pi}{2\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$$

$= \frac{\pi}{2\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$ $\frac{1}{2}$

$\Rightarrow\ 2I = \frac{\pi}{2\sqrt{2}} \left\{ \log\left| \sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right) \right| \right\}_0^{\pi/2}$ $\frac{1}{2}$

$= \frac{\pi}{2\sqrt{2}} \{\log|\sqrt{2}+1| - \log|\sqrt{2}-1|\}$ $\frac{1}{2}$

$\Rightarrow\ I = \frac{\pi}{4\sqrt{2}} \{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)$ or $\frac{\pi}{4\sqrt{2}} \log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ $\frac{1}{2}$

21. The given differential equation can be written as:

$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$ $\frac{1}{2}$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get 1

$$v + x\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$$

$\Rightarrow\ \cot v\ dv = -\frac{1}{x} dx,$ 1

Integrating both sides we get,

$$\left.\begin{array}{l} \log|\sin v| = -\log|x| + \log c \\ \Rightarrow \quad \log|\sin v| = \log\left|\frac{c}{x}\right| \end{array}\right\} \qquad 1$$

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x \cdot \sin\left(\frac{y}{x}\right) = c \qquad \frac{1}{2}$$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x} \cdot y = \frac{-x}{1+\sin x}; \qquad 1\frac{1}{2}$$

$$\text{I.F.} = e^{\int \frac{\cos x}{1+\sin x} dx} = e^{\log(1+\sin x)} = 1 + \sin x \qquad 1$$

$\therefore$ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\, dx + c \qquad 1$$

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{1+\sin x} \qquad \frac{1}{2}$$

22. Lines are perpendicular

$\therefore$ $-3(3\lambda) + 2\lambda(2) + 2(-5) = 0 \Rightarrow \lambda = -2$ 2

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix} = -63 \neq 0 \qquad 1\frac{1}{2}$$

$\therefore$ Lines are not intersecting $\frac{1}{2}$

23. $\vec{a} \cdot \frac{(\vec{b} + \vec{c})}{|\vec{b} + \vec{c}|} = 1$ 1

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2+\lambda)^2 + 36 + 4} \qquad 1\frac{1}{2}$$

$$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ 1

SECTION D

24. $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$ $1\frac{1}{2}$

$A^3 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$ $1\frac{1}{2}$

LHS = $A^3 - 6A^2 + 5A + 11I$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ 1

$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} = O = R.H.S.$

$A^3 - 6A^2 + 5A + 11I = O$, Pre-multiplying by A^{-1}

$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = O \Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$ 1

$\therefore A^{-1} = \begin{bmatrix} -3/11 & 4/11 & 5/11 \\ 9/11 & -1/11 & -4/11 \\ 5/11 & -3/11 & -1/11 \end{bmatrix}$ 1

OR

Let,

$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

System of equation in Matrix form: $A \cdot X = B$

$|A| = 3(2-3) + 2(4+4) + 3(-6-4) = -17 \neq 0$ 1

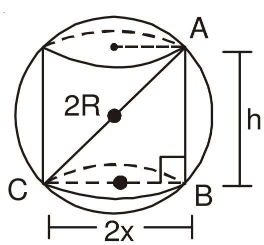
Solution matrix, $X = A^{-1} \cdot B$. 1

$$(\text{adj } A) = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$ 2

$$\therefore \quad A^{-1} = \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$ $\frac{1}{2}$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, y = 2, x = 3$$ $1\frac{1}{2}$

25. Correct Figure 1



ln rt. ΔABC; $4x^2 + h^2 = 4R^2, x^2 = \frac{4R^2 - h^2}{4}$ 1

V(Volume of cylinder) $= \pi x^2 h = \frac{\pi}{4}(4R^2 h - h^3)$ 1

$V'(h) = \frac{\pi}{4}(4R^2 - 3h^2); \; V''(h) = \frac{\pi}{4}(-6h)$ $\frac{1}{2} + \frac{1}{2}$

$V'(h) = 0 \Rightarrow h = \frac{2R}{\sqrt{3}}$ 1

$V''\left(\frac{2R}{\sqrt{3}}\right) = \frac{-6\pi}{4}\left(\frac{2R}{\sqrt{3}}\right) < 0 \Rightarrow$ Volume 'V' is max. $\frac{1}{2}$

for $h = \frac{2R}{\sqrt{3}}$

Max. Volume: $V = \frac{4}{3\sqrt{3}}\pi R^3$ $\frac{1}{2}$

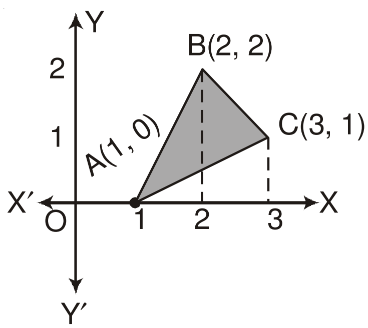
26. E_1 = Event of selecting first bag
E_2 = Event of selecting second bag
A = Event both balls drawn are red. } 1

$$P(E_1) = P(E_2) = \frac{1}{2}; P(A|E_1) = \frac{{}^5C_2}{{}^9C_2} = \frac{20}{72}; P(A|E_2) = \frac{{}^3C_2}{{}^9C_2} = \frac{6}{72}$$ 1+2

$$P(E_2|A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{6}{72}}{\frac{1}{2}\cdot\frac{20}{72} + \frac{1}{2}\cdot\frac{6}{72}} = \frac{6}{26} = \frac{3}{13}$$ $1\frac{1}{2}+\frac{1}{2}$

27. Correct Figure 1



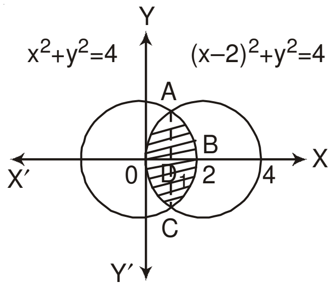
Equation of line AB : $y = 2(x-1)$
Equation of line BC : $y = 4 - x$
Equation of line AC : $y = \frac{1}{2}(x-1)$ } $1\frac{1}{2}$

$$ar(\Delta ABC) = 2\int_1^2 (x-1)\,dx + \int_2^3 (4-x)\,dx - \frac{1}{2}\int_1^3 (x-1)\,dx$$ $1\frac{1}{2}$

$$= (x-1)^2\Big]_1^2 - \frac{1}{2}(4-x)^2\Big]_2^3 - \frac{1}{4}(x-1)^2\Big]_1^3$$ $1\frac{1}{2}$

$$= 1 + \frac{3}{2} - 1 = \frac{3}{2} \text{ sq. units}$$ $\frac{1}{2}$

OR



Correct Figure 1

Getting the point of intersection as x = 1 1

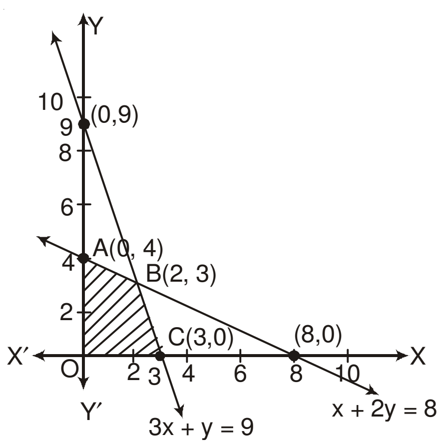
Area (OABCO) = 4 × ar(ABD)

$$= 4\int_1^2 \sqrt{2^2 - x^2}\, dx$$ 2

$$= 4\left\{\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right\}_1^2$$ 1

$$= \left(\frac{8\pi}{3} - 2\sqrt{3}\right)$$ 1

28.



let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:

Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\}$$ $2\frac{1}{2}$

Correct graph. 2

Corner point	Value of z (₹)	
A(0, 4)	200	
B(2, 3)	230 (Max)	$\frac{1}{2}$
C(3, 0)	120	

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$

29. Let $\vec{a}=\hat{i}+\hat{j}-2\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+\hat{k},\ \vec{c}=\hat{i}+2\hat{j}+\hat{k}$

$$(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 0 & 1 & 3\end{vmatrix}=-9\hat{i}-3\hat{j}+\hat{k}$$ 2

Vector equation of plane is:

$$\{\vec{r}-(\hat{i}+\hat{j}-2\hat{k})\}\cdot(-9\hat{i}-3\hat{j}+\hat{k})=0$$

$\Rightarrow\ \vec{r}\cdot(-9\hat{i}-3\hat{j}+\hat{k})+14=0$ 1

Cartesian Equation of plane is: $-9x - 3y + z + 14 = 0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$$-9(x-2)-3(y-3)+(z-7)=0$$

$\Rightarrow\ -9x - 3y + z + 20 = 0$ 1

Distance between parallel planes = $\left|\dfrac{-14+20}{\sqrt{91}}\right|=\dfrac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{2}$ 1

Equation of plane: $\begin{vmatrix}x-2 & y & z-3\\ -1 & 1 & 2\\ 1 & 2 & 1\end{vmatrix}=0$ 1

$\Rightarrow\ -3(x-2)+3y-3(z-3)=0$

$\Rightarrow\ x - y + z - 5 = 0$ 1

General point on line: $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{2}=k$ (say)

is: P(3k + 2, 4k – 1, 2k + 2); Putting in the equation of plane 1

we get, $3k + 2 - 4k + 1 + 2k + 2 = 5 \Rightarrow k = 0$ 1

$\therefore$ Point of intersection is: (2, –1, 2) 1