QUESTION PAPER CODE 30/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. LCM $(x^3y^2, xy^3) = x^3y^3$. 1

2. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore$ $99 = 12 + (n - 1) \times 3$

$\Rightarrow$ $n = 30$ $\frac{1}{2}$

3. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore$ $\frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore$ $ar(\Delta ABC) : ar(\Delta ADE) = 9 : 1$

4. Let the point A be (x, y)

$\therefore$ $\frac{1+x}{2} = 2$ and $\frac{4+y}{2} = -3$ $\frac{1}{2}$

$\Rightarrow$ $x = 3$ and $y = -10$

$\therefore$ Point A is (3, –10) $\frac{1}{2}$

5. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow$ $16 - 4k \geq 0$ $\frac{1}{2}$

$\Rightarrow$ $k \leq 4$ $\frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots = 1 $\frac{1}{2}$

$\Rightarrow$ $\frac{k}{3} = 1 \Rightarrow k = 3$ $\frac{1}{2}$

6. $\tan 2A = \cot (90° - 2A)$

$\therefore \quad 90° - 2A = A - 24°$ $\frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\frac{1}{2}$

SECTION B

7. Required numbers are

14, 21, 28, 35, ..., 98. 1

$98 = 14 + (n - 1) \times 7$ $\frac{1}{2}$

$\Rightarrow \quad n = 13$ $\frac{1}{2}$

OR

Given $S_n = n^2$

$S_1 = a_1 = 1$ $\frac{1}{2}$

$S_2 = a_1 + a_2 = 4$

$\Rightarrow a_2 = 3$ $\frac{1}{2}$

$\therefore d = a_2 - a_1 = 2$ $\frac{1}{2}$

$a_{10} = 1 + 18 = 19$ $\frac{1}{2}$

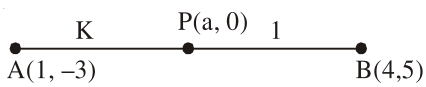
8. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) $= 1 - \frac{1}{4} = \frac{3}{4}$. $\frac{1}{2}$

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$$\therefore \quad a = \frac{4k+1}{k+1}$$

$$0 = \frac{5k-3}{k+1}$$

$\Rightarrow$ $k = \frac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\frac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) $= \frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) $= \frac{3}{6}$ or $\frac{1}{2}$. 1

11. System of equations has infinitely many solutions

$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$ $\frac{1}{2}$

$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6$ or $c = -6$ ...(1) $\frac{1}{2}$

Also $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ ...(2) $\frac{1}{2}$

From equations (1) and (2)

$c = 6$. $\frac{1}{2}$

12. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

$a = 4q + r, 0 \le r < 4$

$\Rightarrow$ $a = 4q$, $a = 4q + 1$, $a = 4q + 2$ and $a = 4q + 3$. 1

Now $a = 4q$ and $a = 4q + 2$ are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. $\frac{1}{2}$

SECTION C

13. Let $p(x) = 3x^3 + 10x^2 - 9x - 4$.

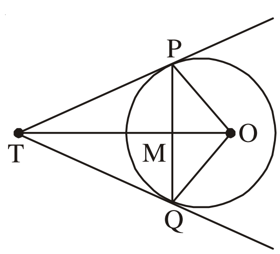
One of the zeroes is 1, therefore dividing p(x) by (x – 1)

$p(x) = (x - 1)(3x^2 + 13x + 4)$ $1\frac{1}{2}$

$= (x - 1)(x + 4)(3x + 1)$ 1

All zeroes are $x = 1$, $x = -4$ and $x = -\frac{1}{3}$. $\frac{1}{2}$

14.



Join OQ, TP = TQ $\therefore$ TM $\perp$ PQ and bisects PQ

Hence PM = 4 cm.

$\therefore$ $OM = \sqrt{25-16} = \sqrt{9} = 3\,\text{cm}$ 1

Let TM = x $\therefore$ $PT^2 = x^2 + 16$ (ΔPMT)

$PT^2 = (x + 3)^2 - 25$ (ΔPOT)

Hence $x^2 + 16 = (x + 3)^2 - 25 = x^2 + 9 + 6x - 25$ 1

$\Rightarrow 6x = 32 \Rightarrow x = \frac{16}{3}$

Hence $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$

$\Rightarrow PT = \frac{20}{3}\,\text{cm}$ 1

15. Let us assume $\frac{2+\sqrt{3}}{5}$ be a rational number.

Let $\frac{2+\sqrt{3}}{5} = \frac{a}{b}$ (b ≠ 0, a and b are integers)

$\Rightarrow \quad \sqrt{3} = \frac{5a-2b}{b}$ 1

$\because$ a, b are integers

$\therefore \quad \frac{5a-2b}{b}$ is a rational number 1

i.e. $\sqrt{3}$ is a rational number

which contradicts the fact that $\sqrt{3}$ is irrational

Therefore is $\frac{2+\sqrt{3}}{5}$ is an irrational number. 1

16. LHS = $\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta = \text{RHS}$ $\frac{1}{2}$

OR

LHS = $\left(1 + \frac{1}{\tan A} - \text{cosec}\,A\right)(1 + \tan A + \sec A)$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)\,(1 + \tan A + \sec A)$ 1

$= \frac{1}{\tan A}[(1+\tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 = \text{RHS}$ 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$ 1

$$= (\sin A + \cos A - 1)(\cos A + \sin A + 1)\cdot\frac{1}{\cos A \sin A}$$

$$= \left[(\sin A + \cos A)^2 - 1\right]\times\frac{1}{\sin A \cos A}$$ 1

$$= (1 + 2\sin A \cos - 1)\times\frac{1}{\sin A \cos A}$$ $\frac{1}{2}$

$$= 2 = \text{RHS}$$ $\frac{1}{2}$

17. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore$ $y = 3x$ ...(1) 1

and $y + 5 = 2(x + 10)$ ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore$ $\frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get $x = 7$, $y = 15$.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

18. Let the required point on y-axis be (0, b) $\frac{1}{2}$

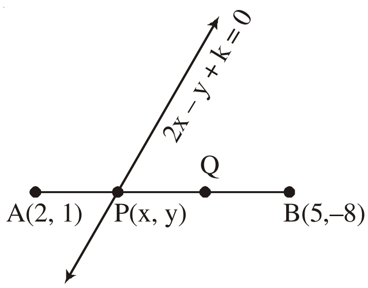
$\therefore$ $(5-0)^2 + (-2-b)^2 = (-3-0)^2 + (2-b)^2$ 1

$\Rightarrow$ $29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow$ $b = -2$ 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2}+\frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on $2x - y + k = 0$

$\Rightarrow$ $6 + 2 + k = 0$

$\Rightarrow$ $k = -8$. 1

19. Modal class is 30-40 $\frac{1}{2}$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 30 + \left(\frac{16-10}{32-10-12}\right) \times 10$ 2

$= 36.$ $\frac{1}{2}$

20. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000 \text{ m}^3$ 1

If 8 cm standing water is needed

then area irrigated = $\frac{6 \times 1.5 \times 5000}{.08} = 562500 \text{ m}^2$. $1+\frac{1}{2}$

21. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{CD}$ ...(1) 1

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

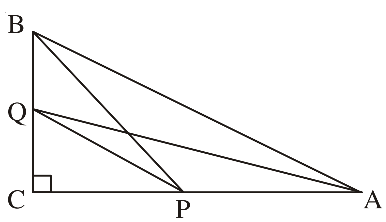
$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$$\frac{AD}{CD} = \frac{CD}{BD}$$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2$. 1

22. $AC = \sqrt{64+36} = 10$ cm.

$\therefore$ Radius of the circle (r) = 5 cm. 1

Area of shaded region = Area of circle – Ar(ABCD) $\frac{1}{2}$

$= 3.14 \times 25 - 6 \times 8$ 1

$= 78.5 - 48$

$= 30.5 \text{ cm}^2$. $\frac{1}{2}$

SECTION D

23. $\sec^2\theta = \left(x + \frac{1}{4x}\right)^2 = x^2 + \frac{1}{16x^2} + \frac{1}{2}$ 1

$\therefore \quad \tan^2\theta = \sec^2\theta - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$ 1

$\Rightarrow \quad \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right)$ or $\left(\frac{1}{4x} - x\right)$ 1

Hence $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$ 1

24. Correct given, to prove, figure, construction $\frac{1}{2} \times 4 = 2$

Correct proof. 2

25. Less than type distribution is as follows

Daily income	**Number of workers**
Less than 220	12
Less than 240	26
Less than 260	34
Less than 280	40
Less than 300	50

Correct Table $1\frac{1}{2}$

Plotting of points (220, 12), (240, 26), (260, 34) (280, 40) and (300, 50) $1\frac{1}{2}$

Joining to get curve 1

OR

Daily expenditure	x_i	No. of households (f_i)	$u_i = \frac{x - 225}{50}$	f_iu_i
100-150	125	4	–2	–8
150-200	175	5	–1	–5
200–250	225	12	0	0
250–300	275	2	1	2
300-350	325	2	2	4
		$\Sigma f_i = 25$		$\Sigma f_iu_i = -7$

Correct Table 2

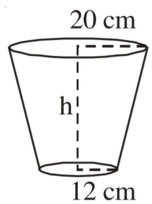
Mean = $225 + 50 \times \left(\frac{-7}{25}\right) = 211$ 2

Mean expenditure on food is ₹ 211.

26. Correct construction of ΔABC. 2

Correct construction of triangle similar to triangle ABC. 2

27.



Volume of the bucket = 12308.8 cm^3

Let $r_1 = 20$ cm, $r_2 = 12$ cm

$\therefore \quad V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \frac{3.14 \times h}{3}(400 + 144 + 240)$ 1

$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784} = 15 \text{ cm}$ 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

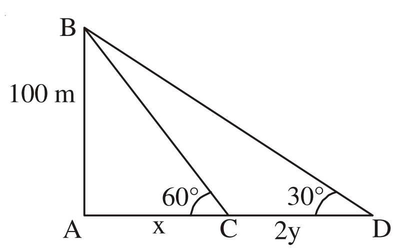
$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l (r_1 + r_2)$

$= 3.14 (144 + 17 \times 32)$

$= 2160.32 \text{ cm}^2$. 1

28. Correct Figure 1



Let the speed of the boat be y m/min

$\therefore$ CD = 2y

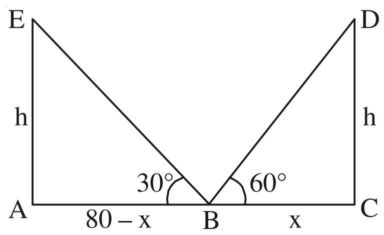
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x + 2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR



Correct Figure 1

Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80-x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1

29. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ 2

$\Rightarrow \quad 4x^2 - 23x + 15 = 0$ $\frac{1}{2}$

$\Rightarrow \quad (4x - 3)(x - 5) = 0$

$x \neq \frac{3}{4} \quad \therefore x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\frac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ...(i) 1

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get $x = 8$, $y = 3$. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

30. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$ 1

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$ 1

Solving to get $d = 2$ $\frac{1}{2}$

and $a = 7$ $\frac{1}{2}$

$\therefore \quad Sn = \frac{n}{2}[14 + (n - 1) \times 2]$

$= n(n + 6)$ or $(n^2 + 6n)$ 1