QUESTION PAPER CODE 65/4/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\cos\theta = \frac{3+12-4}{3\times 7} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\right)$ $\frac{1}{2}+\frac{1}{2}$

OR

$\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1$, length of $x-\text{intercept}=\frac{5}{2}$ 1

2. $\frac{dy}{dx}=\frac{-\sin e^x}{\cos e^x}\cdot e^x$ or $-e^x\cdot\tan e^x$ 1

3. $|A\cdot \text{adj } A| = |A|^n = 4^n$ or 16 or 64 $\frac{1}{2}+\frac{1}{2}$

4. $\frac{dy}{dx}=A\cos x$ $\frac{1}{2}$

$\Rightarrow \frac{dy}{dx}=y\cot x$ $\frac{1}{2}$

SECTION B

5. $I = \tan^{-1}x\cdot\frac{x^2}{2}-\int\frac{x^2}{2(1+x^2)}dx$ 1

$= \frac{x^2}{2}\tan^{-1}x-\frac{1}{2}[x-\tan^{-1}x]+C$ 1

OR

$I = \frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\frac{5}{2}-2x-x^2}}$ $\frac{1}{2}$

$= \frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2-(x+1)^2}}$ 1

$= \frac{1}{\sqrt{2}}\sin^{-1}\left[\frac{(x+1)\sqrt{2}}{\sqrt{7}}\right]+C$ $\frac{1}{2}$

6. I.F. $= e^x$ $\frac{1}{2}$

Solution is $y.e^x = \int (\cos x - \sin x) e^x \, dx + C$ $\frac{1}{2}$

$y.e^x = e^x \cos x + C$ or $y = \cos x + Ce^{-x}$ 1

7. $\int_{-\pi/4}^{0} \tan\left(\frac{\pi}{4} + x\right) dx = \log\left|\sec\left(\frac{\pi}{4} + x\right)\right|_{-\pi/4}^{0}$ $1+\frac{1}{2}$

$= \log\sqrt{2}$ or $\frac{1}{2}\log 2$ $\frac{1}{2}$

8. $a, b \in R \Rightarrow 2a \in R \Rightarrow 2a + b \in R$ $\quad \therefore$ * is binary 1

$a * (b * c) = a * (2b + c) = 2a + 2b + c$

$(a * b) * c = (2a + b) * c = 4a + 2b + c$

In general $a * (b * c) \neq (a * b) * c$ $\quad \therefore$ * is not associative 1

9. Position vector of z $= \dfrac{2(\vec{a} - 3\vec{b}) - 1(3\vec{a} + \vec{b})}{2 - 1}$ 1

$= -\vec{a} - 7\vec{b}$ 1

OR

$\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$ $\frac{1}{2}$

$\vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$ $\frac{1}{2}$

$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = -8 + 3 + 5 = 0$ $\frac{1}{2}$

so $(\vec{a} + \vec{b}) \perp (\vec{a} - \vec{b})$ $\frac{1}{2}$

10. Required probability $= \dfrac{{}^3C_2 \times {}^5C_2}{{}^8C_4}$ 1

$= \dfrac{3}{7}$ 1

OR

$n = 5 \quad p = \frac{1}{3} \quad q = \frac{2}{3}$ $\frac{1}{2}$

$P(X \geq 4) = P(X = 4) + P(X = 5)$

$= 5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5$ 1

$= \frac{11}{243}$ $\frac{1}{2}$

11. P(Problem is solved) = 1 – P(Problem is not solved) $\frac{1}{2}$

$= 1 - \frac{2}{3} \times \frac{4}{5}$ 1

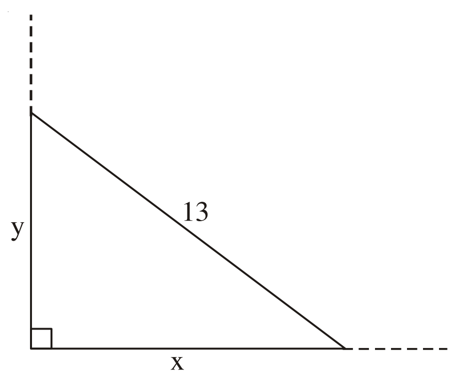
$= \frac{7}{15}$ $\frac{1}{2}$

12. $A + A' = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$ $\frac{1}{2} + \frac{1}{2}$

$(A + A')' = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix} = A + A'$ So A + A′ is symmetric 1

SECTION C

13. Figure $\frac{1}{2}$



$\frac{dx}{dt} = 2 \text{ cm/sec}$ $\frac{1}{2}$

$y = \sqrt{169 - x^2}$ 1

$\frac{dy}{dt} = -\frac{x}{\sqrt{169 - x^2}} \frac{dx}{dt}$ 1

$\left(\frac{dy}{dt}\right)_{x=5} = \frac{-5}{6} \text{ cm/sec}$ 1

Hence height is decreasing at the rate $\frac{5}{6}$ cm / sec

14. LHS becomes $\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{3}{4}$ 1

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}}\right)=\tan^{-1}\frac{56}{33}$$ 1+1

$$= \sin^{-1}\frac{56}{65} = \text{RHS}$$ 1

15. Let $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

$$\text{RHS} = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$$ $\frac{1}{2}$

$$I = \int_0^1 x^2(1-x)^n\,dx$$

$$= \int_0^1 (1-x)^2 x^n dx$$ 1

$$= \int_0^1 [x^n + x^{n+2} - 2x^{n+1}]\,dx$$ $\frac{1}{2}$

$$= \left.\frac{x^{n+1}}{n+1}+\frac{x^{n+3}}{n+3}-\frac{2x^{n+2}}{n+2}\right|_0^1$$ 1

$$= \frac{1}{n+1}+\frac{1}{n+3}-\frac{2}{n+2}$$ $\frac{1}{2}$

16. $\frac{dx}{dt}=\cos t,\quad \frac{dy}{dt}=p\cos pt \;\Rightarrow\; \frac{dy}{dx}=\frac{p\cos pt}{\cos t}$ 1

$$\frac{dy}{dx}=\frac{p\sqrt{1-y^2}}{\sqrt{1-x^2}}$$ 1

$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = p^2(1-y^2)$ differentiating both sides w.r.t x $\frac{1}{2}$

$$\Rightarrow\ 2(1-x^2)\frac{dy}{dx}\frac{d^2y}{dx^2}-2x\left(\frac{dy}{dx}\right)^2=-2p^2y\frac{dy}{dx}$$ 1

$$\Rightarrow\ (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$$ $\frac{1}{2}$

OR

Let $\theta = \cos^{-1} x^2 \Rightarrow x^2 = \cos\theta$ 1

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)$$ 1

$$= \frac{\pi}{4}-\frac{1}{2}\theta$$ 1

$$\therefore\ \frac{dy}{d\theta}=-\frac{1}{2}$$ 1

17. $I = \int\frac{\cos(x+b+a-b)}{\sin(x+b)}dx$ 1

$$= \int[\cot(x+b)\cos(a-b)-\sin(a-b)]\,dx$$ 1

$= \cos(a-b)\log|\sin(x+b)| - x\sin(a-b) + C$ 2

18. Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$$\Rightarrow\ \frac{x_1-1}{x_1-2}=\frac{x_2-1}{x_2-2}$$ $\frac{1}{2}$

$\Rightarrow\ x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$

$\Rightarrow\ x_1 = x_2$ 1

So f is one – one

For range let $f(x) = y$

$$\frac{x-1}{x-2} = y$$ $\frac{1}{2}$

$$x = \frac{2y-1}{y-1}$$ 1

Range of f = R – {1} = co domain B $\frac{1}{2}$

So f is onto.

$$f^{-1}(y) = \frac{2y-1}{y-1} \text{ or } f^{-1}(x) = \frac{2x-1}{x-1}$$ $\frac{1}{2}$

OR

For reflexive 1

For symmetric 1

For transitive

Let (a, b) ∈ S & (b, c) ∈ S

|a – b| = 3m, |b – c| = 3n

a – b = ±3m b – c = ±3n

a –c = 3(±m ± n) ⇒ a – c is divisible by 3 $1\frac{1}{2}$

⇒ |a – c| is divisible by 3

⇒ (a, c) ∈ S

S is transitive

As S is reflexive, symmetric & transitive

∴ S is an equivalence reation. $\frac{1}{2}$

19. Given differential equation can be written as

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$ 1

$$\int \frac{dy}{1+y^2} = \int (x^2+1)\,dx$$ 1

$$\tan^{-1} y = \frac{x^3}{3} + x + C$$ 1

$$x = 0,\ y = 1 \Rightarrow C = \frac{\pi}{4}$$ $\frac{1}{2}$

So particular solution is $\tan^{-1} y = \frac{x^3}{3} + x + \frac{\pi}{4}$ $\frac{1}{2}$

OR

Clearly given differential equation can be written as $\frac{dy}{dx} = \frac{y/x}{1+(y/x)^2}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

Given equation becomes

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$$

$$\int \frac{(1+v^2)}{v^3} dV = -\int \frac{dx}{x}$$ 1

$$-\frac{1}{2v^2} + \log|v| = -\log|x| + C$$ 1

$$-\frac{x^2}{2y^2} + \log|y| = C$$

$$x = 0,\ y = 1 \Rightarrow C = 0$$ $\frac{1}{2}$

So, particular solution is $\log|y| = \frac{x^2}{2y^2}$ $\frac{1}{2}$

20. $R_1 \to R_1 + R_2 + R_3$ & taking 12 + x common

$$(12+x)\begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$ $1+\frac{1}{2}$

$C_2 \to C_2 - C_1 \quad C_3 \to C_3 - C_1$

$$(12+x)\begin{vmatrix} 1 & 0 & 0 \\ 4+x & -2x & 0 \\ 4+x & 0 & -2x \end{vmatrix} = 0$$ $1\frac{1}{2}$

$4x^2 (12 + x) = 0$ $\frac{1}{2}$

$x = 0$ or $x = -12$ $\frac{1}{2}$

21. Required equation of plane is given by

$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 + \lambda[(\vec{r} \cdot (2\hat{i} - \hat{j} - \hat{k}) + 5] = 0$ $\frac{1}{2}$

$\vec{r} \cdot [(1+2\lambda)\hat{i} + (2+\lambda)\hat{j} + (3-\lambda)\hat{k}] = 4 - 5\lambda$ 1

It is perpendicular to $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

So, $5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$ 1

$19\lambda = 7$

$\lambda = \frac{7}{19}$ $\frac{1}{2}$

Required equation is

$\vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) = 41$ 1

22. Here $\overrightarrow{AB} = \hat{i} + (x-2)\hat{j} + 4\hat{k}$, $\overrightarrow{AC} = \hat{i} - 3\hat{k}$, $\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $1\frac{1}{2}$

As A, B, C, D are coplanar

$$\therefore \quad \begin{vmatrix} 1 & x-2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$ 1

$-9 - (2 - x)7 - 4 \times 3 = 0$ 1

$x = 5$ $\frac{1}{2}$

23. Let $u = (\log x)^x \quad v = x^{\log x}$

$$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad 1$$

$$\log u = x \log x$$

$$\frac{1}{u}\frac{du}{dx} = \frac{x}{\log x}\cdot\frac{1}{x} + \log \log x$$

$$\frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log \log x\right] \qquad 1$$

$$\log v = (\log x)^2$$

$$\frac{1}{v}\frac{dv}{dx} = \frac{2\log x}{x}$$

$$\therefore \frac{dv}{dx} = x^{\log x}\left(\frac{2\log x}{x}\right) \qquad 1$$

So $\frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log \log x\right] + 2x^{(\log x - 1)} \cdot \log x$ 1

SECTION D

24. Required equation of line is

$$\vec{r} = 2\hat{i} + 3\hat{j} + 2\hat{k} + \mu(2\hat{i} - 3\hat{j} + 6\hat{k}) \qquad 2$$

$$\vec{a}_1 = -2\hat{i} + 3\hat{j}, \vec{a}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}, \quad \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 2\hat{k} \qquad 1$$

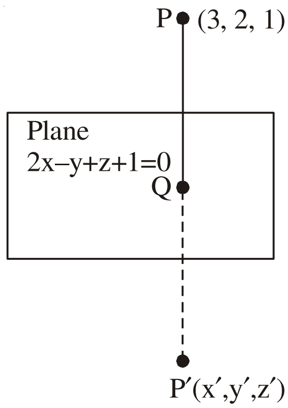
$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix}$$

$$= 6\hat{i} - 20\hat{j} - 12\hat{k} \qquad 2$$

$d = \frac{\sqrt{580}}{7}$ 1

OR



Correct figure 1

Equation of PQ

$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda$ $\frac{1}{2}$

Coodinates of Q $(2\lambda + 3, -\lambda + 2, \lambda + 1)$ $\frac{1}{2}$

As Q lies on plane

$\therefore\ 4\lambda + 6 + \lambda - 2 + \lambda + 1 = -1$

gives, $\lambda = -1$ 1

Coordinates of Q (1, 3, 0) $\frac{1}{2}$

$PQ = \sqrt{6}$ 1

For unique $P'(x', y', z')$

$\frac{x'+3}{2} = 1,\ \frac{y'+2}{2} = 3,\ \frac{z'+1}{2} = 0$ 1

$x' = -1\ y' = 4\ z' = -1$

image is (–1, 4, –1) $\frac{1}{2}$

25. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ we know that A = IA

i.e., $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_1, R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 14 & -25 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_2 \to -(R_2 - 3R_3)$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 1 & -2 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + 4R_2$

$R_3 \to R_3 - 5R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 11 \\ 0 & -1 & 3 \\ -1 & 5 & -13 \end{bmatrix} A$$ 4

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ 1

OR

$|A| = 67 \neq 0$ $\therefore X = A^{-1}B$ $1+\frac{1}{2}$

$$\text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ 2

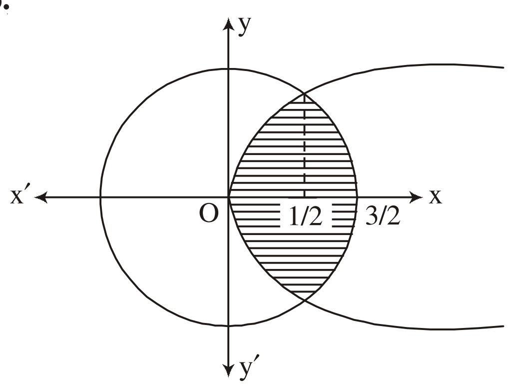
$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ $\frac{1}{2}$

$$\text{So } X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$ $1\frac{1}{2}$

$x = 3,\ y = -2,\ z = 1$ $\frac{1}{2}$

26.



Correct Figure 1

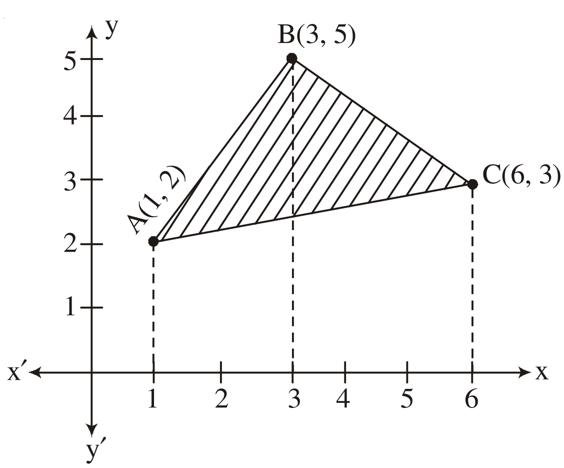
x coordinate of Point of intersection $= \frac{1}{2}$ 1

Required Area $= 2\left[\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{3/2} \sqrt{\left(\frac{3}{2}\right)^2 - x^2}\,dx\right]$ 2

$= 2\left[\frac{4}{3}x^{3/2}\Big|_0^{\frac{1}{2}} + \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Big|_{\frac{1}{2}}^{3/2}\right]$ 1

$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$ 1

OR



Correct figure 1

Correct points of intersection $1\frac{1}{2}$

Required Area $= \int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{21-2x}{3}dx - \int_1^6 \frac{x+9}{5}dx$ 2

$$= \left(\frac{3x^2}{4}+\frac{x}{2}\right)\Bigg|_1^3 + \left(7x-\frac{x^2}{3}\right)\Bigg|_3^6 - \left(\frac{x^2}{10}+\frac{9x}{5}\right)\Bigg|_1^6$$ 1

$$= 7+12-\frac{25}{2}$$

$$= \frac{13}{2}$$ $\frac{1}{2}$

27. E_1: Selected person is cyclist
E_2: Selected person is scooter driver
E_3: Selected person is car driver
A: insured person met with an accident 1

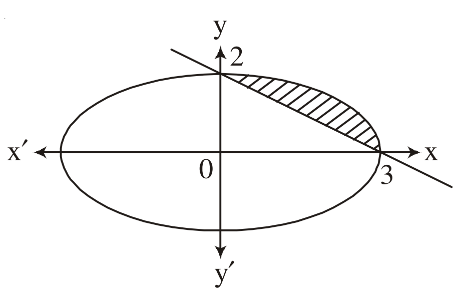
$$P(E_1)=\frac{3}{18},\ P(E_2)=\frac{6}{18},\ P(E_3)=\frac{9}{18}$$
$$P(A|E_1)=0.3,\ P(A|E_2)=0.05,\ P(A|E_3)=0.02$$ 2

$$P(E_1|A)=\frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1)+P(E_2)\,P(A|E_2)+P(E_3)\,P(A|E_3)}$$

$$= \frac{\frac{3}{18}\times\frac{30}{100}}{\frac{3}{18}\times\frac{30}{100}+\frac{6}{18}\times\frac{5}{100}+\frac{9}{18}\times\frac{2}{100}}$$ 2

$$= \frac{90}{138} \text{ or } \frac{15}{23}$$ 1

28.



Correct Figure 1

Point of intersection are (3, 0) & (0, 2) 1

$$\text{Required Area} = \frac{2}{3}\int_0^3 \sqrt{(9-x^2)}\,dx - \frac{2}{3}\int_0^3 (3-x)\,dx$$ 2

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}+\frac{(3-x)^2}{2}\right]_0^3$$ 1

$$= \frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]$$

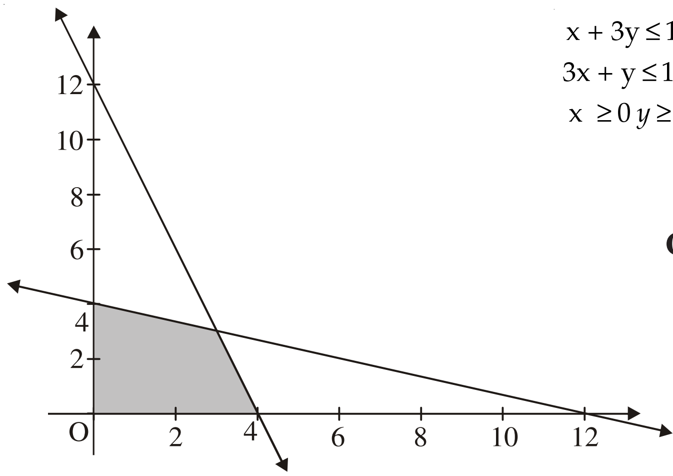
$$= \frac{3\pi}{2}-3$$ 1

29. Let number of packages of nuts = x units

and number of packages of bolts = y units

Our L.P.P is

Maximize profit Z = 35x + 14y $\frac{1}{2}$



$$\left.\begin{array}{l} x + 3y \le 12 \\ 3x + y \le 12 \\ x \ge 0\ y \ge 0 \end{array}\right\}$$ $2\frac{1}{2}$

Correct figure 2

Corner points	**Value of Z**		
(4, 0)	140		
(3, 3)	147	← maximum	$\frac{1}{2}$
(0, 4)	56		

Maximum profit = ₹ 147 when x = 3, y = 3 $\frac{1}{2}$