

QUESTION PAPER CODE 65/3/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

$$1. \text{ DRs are } (6, 2, 3) \therefore \text{ DC's are } \left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right) \quad \frac{1}{2} + \frac{1}{2}$$

OR

$$\frac{x-1}{-3} = \frac{y-7}{p} = \frac{z-3}{2}; \frac{x-1}{-3p} = \frac{y-5}{1} = \frac{z-6}{-5} \quad \frac{1}{2}$$

$$\Rightarrow 9p + p - 10 = 0 \Rightarrow p = 1 \quad \frac{1}{2}$$

$$2. \frac{dy}{dx} - \frac{2}{x} \cdot y = 2x \quad \frac{1}{2}$$

$$\Rightarrow \text{I.F.} = e^{-2 \log x} = x^{-2} = \frac{1}{x^2} \quad \frac{1}{2}$$

$$3. |-2A| = (-2)^3 \cdot |A| \quad \frac{1}{2}$$

$$= -8 \times 4 = -32 \quad \frac{1}{2}$$

$$4. y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0 \quad \frac{1}{2} + \frac{1}{2}$$

SECTION B

$$5. B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix} \quad 1$$

$$= \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix} \quad 1$$

$$6. \int_a^b \frac{\log x}{x} dx = \left[\frac{1}{2} (\log x)^2 \right]_a^b \quad 1$$

$$= \frac{1}{2} [(\log b)^2 - (\log a)^2] \quad 1$$

$$7. \quad y^2 = m(a^2 - x^2) \Rightarrow 2y \frac{dy}{dx} = -2mx \quad \frac{1}{2}$$

$$\text{or } y \frac{dy}{dx} = -mx \quad \dots(i)$$

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m \quad \dots(ii) \quad \frac{1}{2}$$

$$\text{form (i) and (ii) we get } y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x} \frac{dy}{dx} \quad 1$$

$$\text{or } xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^2 - y \frac{dy}{dx} = 0$$

$$8. \quad \int \frac{\sin(x-a)}{\sin(x+a)} dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)} dx \quad \frac{1}{2}$$

$$= \int \left[\frac{\sin(x+a) \cdot \cos 2a}{\sin(x+a)} - \frac{\cos(x+a) \sin 2a}{\sin(x+a)} \right] dx \quad \frac{1}{2}$$

$$= x \cdot \cos 2a - \sin 2a \cdot \log |\sin(x+a)| + c \quad \frac{1}{2} + \frac{1}{2}$$

OR

$$\int (\log x)^2 \cdot 1 dx = (\log x)^2 \cdot x - \int 2 \cdot \log x \cdot \frac{1}{x} \cdot x dx \quad 1$$

$$= x \cdot (\log x)^2 - \left\{ \log x \cdot 2x - \int \frac{1}{x} \cdot 2x dx \right\} \quad \frac{1}{2}$$

$$= x(\log x)^2 - 2x \log x + 2x + c \quad \frac{1}{2}$$

$$9. \quad \text{A vector perpendicular to both } \vec{a} \text{ and } \vec{b} = \vec{a} \times \vec{b} = 19\hat{j} + 19\hat{k} \text{ or } \hat{j} + \hat{k} \quad 1$$

$$\therefore \text{Unit vector perpendicular to both } \vec{a} \text{ and } \vec{b} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}) \quad 1$$

OR

$$\text{Let } \vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\vec{a}, \vec{b}, \vec{c} \text{ are coplanar if } \vec{a} \cdot \vec{b} \times \vec{c} = 0 \quad \frac{1}{2}$$

$$\vec{a} \cdot \vec{b} \times \vec{c} = \left. \begin{array}{l} \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3) \\ = 3 - 12 + 9 = 0 \end{array} \right\} 1 + \frac{1}{2}$$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$$10. \left. \begin{array}{l} A = \{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\} \\ B = \{(S, F, M), (M, F, S)\} \end{array} \right\} 1$$

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{2/6}{4/6} = \frac{1}{2} \quad 1$$

$$11. \text{ Given } 2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$$

$$\text{Let } P(X = x_3) = k, \text{ then } P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3} \text{ and } P(X = x_4) = \frac{k}{5} \quad \frac{1}{2}$$

$$\therefore k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61} \quad 1$$

\therefore Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

1/2

OR

$$(i) P(\text{at least 4 heads}) = P(r \geq 4) = P(4) + P(5)$$

$$= {}^5C_4 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 + {}^5C_5 \left(\frac{1}{2}\right)^5 = 6 \left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16} \quad 1$$

$$(ii) P(\text{at most 4 heads}) = P(r \leq 4) = 1 - P(5)$$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32} \quad 1$$

12. Let $e \in \mathbb{R}$ be the identity element.

$$\text{then } a * e = e * a = a \quad 1$$

$$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0. \quad 1$$

\therefore Identity element is $0 \in \mathbb{R}$

SECTION C

$$13. \tan(\sec^{-1} \frac{1}{x}) = \sin(\tan^{-1} 2) \Rightarrow \tan\left(\tan^{-1} \frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1} \frac{2}{\sqrt{5}}\right) \quad 2$$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5} \quad 1$$

$$\Rightarrow 9x^2 = 5 \Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}, \{x > 0\} \quad 1$$

$$14. e^y \cdot (x+1) = 1 \Rightarrow e^y \cdot 1 + (x+1)e^y \cdot \frac{dy}{dx} = 0 \quad 1$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1} \quad 1$$

$$\Rightarrow \frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2} \quad 1$$

$$\therefore \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 \quad 1$$

OR

$$y = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+(2^x)^2}\right) = \sin^{-1}\left(\frac{2t}{1+t^2}\right), \text{ where } t = 2^x \quad 1$$

$$\Rightarrow y = 2 \tan^{-1} t \quad 1$$

$$\frac{dy}{dt} = \frac{2}{1+t^2} \text{ and } \frac{dt}{dx} = 2^x \cdot \log 2. \quad 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{1+t^2} \cdot 2^x \cdot \log 2 = \frac{2^{x+1} \cdot \log 2}{1+4^x} \quad 1$$

15. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$$\Rightarrow f'(x) = 12x^2 - 12x - 72 = 12(x-3)(x+2) \quad 1$$

$$f'(x) = 0 \Rightarrow x = -2, x = 3 \quad 1$$

\therefore possible intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

$$f'(x) < 0 \text{ in } (-2, 3)$$

and $f'(x) > 0$ in $(-\infty, -2)$ and $(3, \infty)$

$\Rightarrow f(x)$ is strictly increasing in $(-\infty, -2), (3, \infty)$ or $(-\infty, 2], [3, \infty)$ 1

and strictly decreasing in $(-2, 3)$ or $[-2, 3]$ $\frac{1}{2}$

16. (i) For $a \in \mathbb{Z}, (a, a) \in \mathbb{R} \because a - a = 0$ is divisible by 2

$\therefore \mathbb{R}$ is reflexive $\dots(i)$ 1

Let $(a, b) \in \mathbb{R}$ for $a, b \in \mathbb{Z}$, then $a - b$ is divisible by 2

$\Rightarrow (b - a)$ is also divisible by 2

$\therefore (b, a) \in \mathbb{R} \Rightarrow \mathbb{R}$ is symmetric $\dots(ii)$ 1

For $a, b, c, \in \mathbb{Z}$, Let $(a, b) \in \mathbb{R}$ and $(b, c) \in \mathbb{R}$

$\therefore a - b = 2p, p \in \mathbb{Z}$, and $b - c = 2q, q \in \mathbb{Z}$,

adding, $a - c = 2(p + q) \Rightarrow (a - c)$ is divisible by 2

$\Rightarrow (a, c) \in \mathbb{R}$, so \mathbb{R} is transitive $\dots(iii)$ $\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow \mathbb{R}$ is an equivalence relation. $\frac{1}{2}$

OR

$$f \circ f(x) = f\left(\frac{4x+3}{6x-4}\right) \quad 1$$

$$= \frac{4\left[\frac{4x+3}{6x-4}\right] + 3}{6\left[\frac{4x+3}{6x-4}\right] - 4} \quad 1$$

$$\Rightarrow \text{f of}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x \quad 1$$

$$\text{Since } \text{f of}(x) = x \Rightarrow \text{f of} = I \Rightarrow f^{-1} = f \quad 1$$

17. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$$R_1 \rightarrow R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} \quad 1$$

$$\therefore \Delta = -2 \begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b} \begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix} \quad 1 + \frac{1}{2}$$

$$C_2 \rightarrow C_2 - cC_3$$

$$\Rightarrow \Delta = -\frac{2}{b} \begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix} \quad 1$$

$$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc. \quad \frac{1}{2}$$

18. $y = (\sec^{-1} x)^2, x > 0$

$$\frac{dy}{dx} = 2 \sec^{-1} x \cdot \frac{1}{x\sqrt{x^2-1}} \quad 1$$

$$\Rightarrow x\sqrt{x^2-1} \frac{dy}{dx} = 2\sqrt{y} \quad \frac{1}{2}$$

squaring both sides, we get

$$x^2(x^2-1) \left(\frac{dy}{dx} \right)^2 = 4y \quad \text{or} \quad (x^4-x^2) \left(\frac{dy}{dx} \right)^2 = 4y \quad \frac{1}{2}$$

differentiating w.r.t. x.

$$(x^4-x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3-2x) \left(\frac{dy}{dx} \right)^2 = 4 \cdot \frac{dy}{dx} \quad 1$$

$$\Rightarrow x^2(x^2 - 1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0 \quad 1$$

$$19. \text{ RHS} = \int_a^b f(a+b-x) dx = -\int_b^a f(t) dt, \text{ where } a+b-x=t, dx = -dt \quad \frac{1}{2}$$

$$= \int_a^b f(t) dt = \int_a^b f(x) dx = \text{LHS} \quad \frac{1}{2}$$

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots(i) \quad \frac{1}{2}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)} + \sqrt{\sin(\pi/2-x)}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(ii) \quad \frac{1}{2}$$

$$\text{adding (i) and (ii) to get } 2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx = x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \pi/6. \quad \frac{1}{2}$$

$$\Rightarrow I = \frac{\pi}{12} \quad \frac{1}{2}$$

$$20. I = \int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

$$\text{Put } \sin^2 x = t \Rightarrow \sin 2x dx = dt \quad \frac{1}{2}$$

$$\therefore I = \int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3} \right) dt \quad \frac{1}{2}$$

$$= \frac{1}{2} \log|t+1| - \frac{1}{2} \log|t+3| + c \quad \frac{1}{2}$$

$$= \frac{1}{2} \log(\sin^2 x + 1) - \frac{1}{2} \log(\sin^2 x + 3) + c. \quad \frac{1}{2}$$

21. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3} \quad \frac{1}{2}$$

lines are perpendicular to each other,

$$\Rightarrow (5\lambda + 2) \cdot 1 + (-5)(2\lambda) + 1(3) = 0 \quad \frac{1}{2}$$

$$-5\lambda + 5 = 0 \Rightarrow \lambda = 1 \quad \frac{1}{2}$$

$$\therefore \text{lines are } \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-1}{3} \quad \frac{1}{2}$$

$$\text{Shortest distance between these lines} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{\left| \left(5\hat{i} + \frac{5}{2}\hat{j} \right) \cdot (-17\hat{i} - 20\hat{j} + 19\hat{k}) \right|}{|\vec{b}_1 \times \vec{b}_2|} \quad 1$$

$$= \frac{135}{|\vec{b}_1 \times \vec{b}_2|} \neq 0 \quad \frac{1}{2}$$

\therefore lines are not intersecting. $\frac{1}{2}$

$$22. \text{ Given } \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} \therefore \vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a} \quad \dots(i) \quad \frac{1}{2}$$

$$\vec{b} \perp \vec{c} \Rightarrow \vec{b} \cdot \vec{c} = 0 \quad \dots(ii) \quad \frac{1}{2}$$

$$(|3\vec{a} - 2\vec{b} + 2\vec{c}|)^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\vec{a} \cdot \vec{b} - 8\vec{b} \cdot \vec{c} + 12\vec{a} \cdot \vec{c} \quad 1$$

$$= 9(1)^2 + 4(2)^2 + 4(3)^2 \quad \text{[using (i) and (ii)]} \quad 1$$

$$= 9 + 16 + 36 = 61$$

$$\Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61} \quad 1$$

$$23. \frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}} \quad \frac{1}{2}$$

$$\text{Put } y/x = v \text{ so that } \frac{dy}{dx} = v + x \frac{dv}{dx} \quad \frac{1}{2}$$

$$\therefore v + x \frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x \frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$$

$$\Rightarrow \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2} dv - \frac{1}{2} \int \frac{2v}{1+v^2} dv = \int \frac{dx}{x} \quad 1 + \frac{1}{2}$$

$$\Rightarrow \tan^{-1} v = \frac{1}{2} \log |1+v^2| + \log |x| + c \quad 1$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) = \frac{1}{2} \log \left| \frac{x^2 + y^2}{x^2} \right| + \log |x| + c \quad 1$$

$$\text{or } \tan^{-1} \left(\frac{y}{x} \right) = \frac{1}{2} \log |x^2 + y^2| + c$$

OR

$$(1+x^2)dy + 2xy dx = \cot x \cdot dx.$$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\cot x}{1+x^2} \quad 1$$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2) \quad 1$$

$$\therefore \text{Solution is, } y \cdot (1+x^2) = \int \cot x dx = \log |\sin x| + c \quad 1+1$$

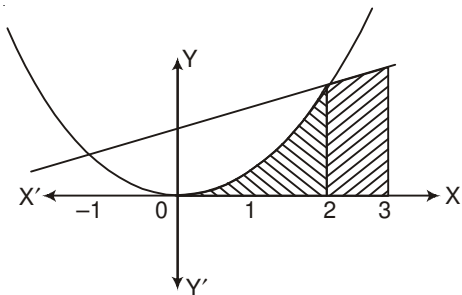
$$\text{or } y = \frac{1}{1+x^2} \cdot \log |\sin x| + \frac{c}{1+x^2}$$

SECTION D

24.

$$\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x+2, -1 \leq x \leq 3\}$$

Correct Figure 1



$$\text{Area} = \int_{-1}^2 x^2 dx + \int_2^3 (x+2) dx \quad 2$$

$$= \left[\frac{x^3}{3} \right]_{-1}^2 + \left[\frac{(x+2)^2}{2} \right]_2^3 \quad 2$$

$$= 3 + \frac{9}{2} = \frac{15}{2} \quad 1$$

OR

$$I = \lim_{h \rightarrow 0} h[f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \quad 1$$

$$\text{where } h = \frac{3}{n} \text{ or } nh = 3 \quad \frac{1}{2}$$

$$= \lim_{h \rightarrow 0} h[(2+e^2) + (2+h+e^{2+2h}) + (2+2h+e^{2+4h}) + \dots + (2+(n-1)h) + e^{2+(n-1)h}] \quad 1$$

$$= \lim_{h \rightarrow 0} h \left[2n + h \cdot \frac{n(n-1)}{2} \right] + \lim_{h \rightarrow 0} h \cdot e^2 [1 + e^{2h} + e^{4h} + \dots + e^{2(n-1)h}] \quad 1$$

$$= \lim_{h \rightarrow 0} \left[2nh + \frac{nh(nh-h)}{2} \right] + \lim_{h \rightarrow 0} h \cdot \frac{e^2}{2h} \cdot \frac{e^{2nh} - 1}{e^{2h} - 1} \quad 1 \frac{1}{2}$$

$$= 6 + \frac{9}{2} + \frac{e^2(e^6 - 1)}{2} = \frac{21}{2} + \frac{e^2(e^6 - 1)}{2} \quad 1$$

25. $p = (\text{prob. of doublet}) = 1/6 \quad \therefore q = 5/6 \quad 1$

$$P(X) \begin{array}{c|c|c|c|c} 0 & 1 & 2 & 3 & 4 \\ \hline \left(\frac{5}{6}\right)^4 & 4 \cdot \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} & 6 \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2 & 4 \cdot \frac{5}{6} \left(\frac{1}{6}\right)^3 & \left(\frac{1}{6}\right)^4 \\ \hline = \frac{625}{1296} & = \frac{500}{1296} & \frac{150}{1296} & \frac{20}{1296} & \frac{1}{1296} \end{array} \quad 1$$

$$\begin{array}{c|c|c|c|c} XP(X) & 0 & \frac{500}{1296} & \frac{300}{1296} & \frac{60}{1296} & \frac{4}{1296} \\ \hline X^2P(X) & 0 & \frac{500}{1296} & \frac{600}{1296} & \frac{180}{1296} & \frac{16}{1296} \end{array} \quad 1$$

$$\text{Mean} = \sum XP(X) = \frac{864}{1296} = \frac{2}{3} \quad 1$$

$$\text{Variance} = \sum X^2 \cdot P(X) - [\sum X \cdot P(X)]^2 = 1 - \frac{4}{9} = \frac{5}{9} \quad 1$$

26. Equation of plane passing through (2, 5, -3), (-2, -3, 5) and (5, 3, -3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0 \quad 1$$

$$\Rightarrow 16(x-2) + 24(y-5) + 32(z+3) = 0$$

$$\text{i.e. } 2x + 3y + 4z - 7 = 0 \quad \dots(\text{i})$$

which in vector form can be written as $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 7$

Equation of line passing through (3, 1, 5) and (-1, -3, -1) is

$$\frac{x-3}{4} = \frac{y-1}{4} = \frac{z-5}{6} \quad \text{or} \quad \frac{x-3}{2} = \frac{y-1}{2} = \frac{z-5}{3} \quad \dots(\text{ii})$$

Any point on (ii) is $(2\lambda + 3, 2\lambda + 1, 3\lambda + 5)$

If this is point of intersection with plane (i), then

$$2(2\lambda + 3) + 3(2\lambda + 1) + 4(3\lambda + 5) - 7 = 0$$

$$22\lambda + 22 = 0 \Rightarrow \lambda = -1$$

\therefore Point of intersection is (1, -1, 2)

OR

Equation of plane through the intersection of planes

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$, is

$$[\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda[\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4] = 0$$

$$\Rightarrow \vec{r} \cdot [(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}] - 1 + 4\lambda = 0 \quad \dots(\text{i})$$

$$\text{Plane (i) is } \parallel \text{ to x-axis} \Rightarrow 1 + 2\lambda = 0 \Rightarrow \lambda = \frac{-1}{2}$$

$$\therefore \text{Equation of plane is } \vec{r} \cdot \left(-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \right) - 3 = 0$$

$$\text{or } \vec{r} \cdot (-\hat{j} + 3\hat{k}) - 6 = 0$$

Distance of this plane from x-axis

$$= \frac{|-6|}{\sqrt{(-1)^2 + (3)^2}} = \frac{6}{\sqrt{10}} \text{ units}$$

27. Let Given surface area of open cylinder be S.

$$\text{Then } S = 2\pi rh + \pi r^2$$

$$\Rightarrow h = \frac{S - \pi r^2}{2\pi r} \quad 1$$

$$\text{Volume } V = \pi r^2 h \quad \frac{1}{2}$$

$$V = \pi r^2 \left[\frac{S - \pi r^2}{2\pi r} \right] = \frac{1}{2} [Sr - \pi r^3] \quad 1$$

$$\frac{dV}{dr} = \frac{1}{2} [S - 3\pi r^2] \quad \frac{1}{2}$$

$$\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2 \quad \text{or} \quad 2\pi rh + \pi r^2 = 3\pi r^2 \quad 1$$

$$\Rightarrow 2\pi rh = 2\pi r^2 \quad \Rightarrow h = r \quad 1$$

$$\frac{d^2V}{dr^2} = -6\pi r < 0 \quad 1$$

\therefore For volume to be maximum, height = radius

28. $|A| = 1(7) - 1(-3) + 1(-1) = 9 \quad 1$

$$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \quad 2$$

$$\Rightarrow A^{-1} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \quad \frac{1}{2}$$

$$\text{Given equations can be written as } \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\text{or } AX = B \Rightarrow X = A^{-1}B \quad \frac{1}{2}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

 $\frac{1}{2}$

$$\therefore x = 1, y = 2, z = 3$$

 $\frac{1}{2}$

OR

$$\text{Let: } \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

1

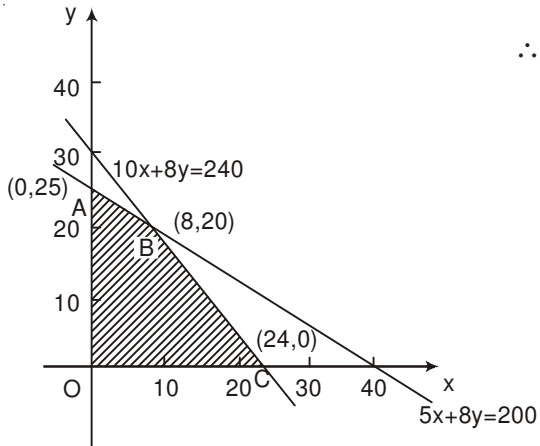
$$\left. \begin{aligned} R_1 &\leftrightarrow R_3 \begin{bmatrix} 3 & 7 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \\ R_1 &\rightarrow R_1 - R_3 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \\ R_2 &\rightarrow R_2 - R_3 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A \\ R_3 &\rightarrow R_3 - 2R_1 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A \\ R_3 &\rightarrow R_3 + 5R_2 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ -2 & 5 & -2 \end{bmatrix} A \\ R_1 &\rightarrow R_1 - 4R_2 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A \\ R_3 &\rightarrow -R_3 \\ R_1 &\rightarrow R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A \end{aligned} \right\}$$

4

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$

1

29.

Let number of Souvenirs of type A be x , and that of type B be y .
 \therefore L.P.P is maximise $P = 50x + 60y$
such that $5x + 8y \leq 200$ $10x + 8y \leq 240$ $x, y \geq 0$ $\frac{1}{2}$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

 \therefore Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.

1