QUESTION PAPER CODE 430/5/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. If (3, – 6) is the mid-point of the line segment joining (0, 0) and (x, y), then the point (x, y) is

(A) (–3, 6) **(B) (6, – 6)** **(C) (6, – 12)** **(D) $(\frac{3}{2}, -3)$**

Sol. (C) (6, –12) 1

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2. In the given circle in Figure-1, number of tangents parallel to tangent PQ is

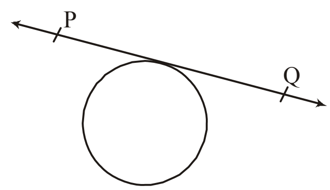

Fig. 1

(A) 0 **(B) many** **(C) 2** **(D) 1**

Sol. (D) 1 1

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3. The discriminant of the quadratic equation $4x^2 - 6x + 3 = 0$ is

(A) 12 **(B) 84** **(C) $2\sqrt{3}$** **(D) – 12**

Sol. (D) –12 1

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4. For the following frequency distribution:

Class:	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	8	10	19	25	8

The upper limit of median class is

(A) 15 **(B) 10** **(C) 20** **(D) 25**

Sol. (A) 15 1

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5. If $\cos A = \frac{\sqrt{3}}{2}$, $0° < A < 90°$, then A is equal to

(A) $\frac{\sqrt{3}}{2}$ **(B) 30°** **(C) 60°** **(D) 1**

Sol. (B) 30° 1

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6. The probability of an impossible event is

(A) 1 **(B) $\frac{1}{2}$** **(C) not defined** **(D) 0**

Sol. (D) 0 1

7. If a pair of linear equations is consistent, then the lines represented by them are

(A) parallel **(B) intersecting or coincident**

(C) always coincident **(D) always intersecting**

Sol. (B) Intersecting or coincident. 1

8. The distance between the points (3, – 2) and (– 3, 2) is

(A) $\sqrt{52}$ units **(B) $4\sqrt{10}$ units** **(C) $2\sqrt{10}$ units** **(D) 40 units**

Sol. (A) $\sqrt{52}$ units 1

9. 180 can be expressed as a product of its prime factors as

(A) $10 \times 2 \times 3^2$ **(B) $2^{25} \times 4 \times 3$** **(C) $2^2 \times 3^2 \times 5$** **(D) $4 \times 9 \times 5$**

Sol. (C) $2^2 \times 3^2 \times 5$ 1

10. The total surface area of a frustum-shaped glass tumbler is ($r_1 > r_2$)

(A) $\pi r_1 l + \pi r_2 l$ **(B) $\pi l (r_1 + r_2) + \pi r_2^2$**

(C) $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$ **(D) $\sqrt{h^2 + (r_1 - r_2)^2}$**

Sol. (B) $\pi l (r_1 + r_2) + \pi r_2^2$ 1

Fill in the blank in question number 11 to 15

11. If 2 is a zero of the polynomial $ax^2 - 2x$, then the value of 'a' is _______.

Sol. 1 1

12. If the radii of two spheres are in the ratio 2 : 3, then the ratio of their respective volumers is _______.

Sol. 8/27 or 8 : 27 1

13. A line intersecting a circle in two points is called a _______.

Sol. Secant 1

14. If ar (Δ PQR) is zero, then the points P, Q and R are _______.

Sol. Collinear 1

15. All squares are _______ (congruent/similar).

Sol. Similar 1

Answer the following question number 16 to 20

16. A coin is tossed twice. Find the probability of getting head both the times.

Sol. Total outcomes = 4 $\frac{1}{2}$

P(getting head both the times) = $\frac{1}{4}$ $\frac{1}{2}$

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17. Find the radius of the sphere whose surface area is 36π cm^2.

Sol. $4\pi r^2 = 36\pi$ $\frac{1}{2}$

$r = 3$ cm $\frac{1}{2}$

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18. Find the value of x so that – 6, x, 8 are in A.P.

Sol. $x + 6 = 8 - x$ $\frac{1}{2}$

$\boxed{x = 1}$ $\frac{1}{2}$

OR

Find the 11th term of the A.P. – 27, – 22, –17, –12, ...

Sol. $a = -27,\ d = 5$ $\frac{1}{2}$

$a_{11} = -27 + 50 = 23$ $\frac{1}{2}$

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19. In Figure-2, the angle of elevation of the top of a tower AC from a point B on the ground is 60°. If the height of the tower is 20 m, find the distance of the point from the foot of the tower.

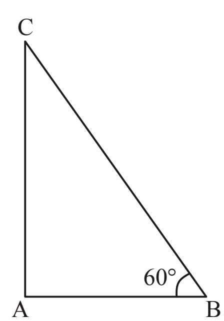

Fig. 2

Sol. $\frac{AC}{AB} = \tan 60°$ $\frac{1}{2}$

$\frac{20}{AB} = \sqrt{3}$

$AB = \frac{20\sqrt{3}}{3}$ or $AB = \frac{20}{\sqrt{3}}$ $\frac{1}{2}$

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20. Evaluate:

tan 40° × tan 50°

Sol. $\tan 40° \times \cot 40°$ $\frac{1}{2}$

$= 1$ $\frac{1}{2}$

OR

If cos A = sin 42°, then find the value of A.

Sol. $\cos A = \sin (90° - 48°)$ $\frac{1}{2}$

$= \cos 48°$

$\Rightarrow \boxed{A = 48°}$ $\frac{1}{2}$

SECTION B

Question number 21 to 26 carry 2 mark each.

21. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$, $0 < A + B \le 90°$, $A > B$, then find the value of A and B.

Sol. $A + B = 60°$ ...(i) 1

$A - B = 30°$ ...(ii) $\frac{1}{2}$

From (i) and (ii)

$\left.\begin{array}{l} A = 45° \\ B = 15° \end{array}\right]$ $\frac{1}{2}$

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22. A letter is selected at random from the set of English alphabets. What is the probability that it is a vowel?

Sol. Total outcomes = 26 1

P(getting vowel) = $\frac{5}{26}$ 1

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23. Solve for x:

$$\sqrt{3}x^2 + 14x - 5\sqrt{3} = 0$$

Sol. $\sqrt{3}x^2 + 15x - x - 5\sqrt{3} = 0$ 1

$[x + 5\sqrt{3}][\sqrt{3}x - 1] = 0$ $\frac{1}{2}$

$x = -5\sqrt{3}$ or $x = \frac{1}{\sqrt{3}}$ $\frac{1}{2}$

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24. Find the mean for the following distribution:

Classes:	5 – 15	15 – 35	25 – 35	35 – 45
Frequency:	2	4	3	1

Sol.

Classes	Freq.	Mid value = x	f × x
5-15	2	10	20
15-25	4	20	80
25-35	3	30	90
35-45	1	40	40
	$\Sigma f = 10$		$\Sigma fx = 230$

Correct table 1

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ $\frac{1}{2}$

$= \frac{230}{10} = 23$ $\frac{1}{2}$

OR

The following distribution shows the transport expenditure of 100 employees:

Expenditure (in (₹):	200 – 400	400 – 600	600 – 800	800 – 1000	1000 – 1200
Number of employees:	21	25	19	23	12

Find the mode of the distribution.

Sol. Modal class = 400 – 600 $\frac{1}{2}$

Mode = $1 + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$ $\frac{1}{2}$

$= 400 + \left[\frac{25 - 21}{50 - 21 - 19}\right] \times 200$ $\frac{1}{2}$

= 400 + 80 = 480 $\frac{1}{2}$

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25. Check whether 6^n can end with the digit '0' (zero) for any natural number n.

Sol. $6^n = (2 \times 3)^n = 2^n \times 3^n$ 1

It is not in form of $2^n \times 5^m$ $\frac{1}{2}$

$\therefore$ 6^n can't end with digit '0' $\frac{1}{2}$

OR

Find the LCM of 150 and 200.

Sol. $150 = 2 \times 3 \times 5^2$ $\frac{1}{2}$

$200 = 2^3 \times 5^2$ $\frac{1}{2}$

$\text{LCM} = 2^3 \times 5^2 \times 3$ $\frac{1}{2}$

$= 600$ $\frac{1}{2}$

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26. In Figure-3, ΔABC and ΔXYZ are shown. If AB = 3 cm BC = 6 cm, AC = $2\sqrt{3}$ cm, ∠A = 80°, ∠B = 60°, XY = $4\sqrt{3}$ cm YZ = 12 cm and XZ = 6 cm, then find the value of ∠Y.

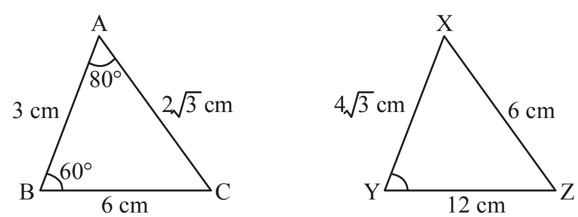

Figure 3

Sol. $\because \frac{AB}{XZ} = \frac{BC}{YZ} = \frac{AC}{XY} = \frac{1}{2}$ 1

$\therefore$ ΔABC ~ ΔXZY $\frac{1}{2}$

∠C = ∠Y = 40° $\frac{1}{2}$

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. Construct a triangle with its sides 4 cm, 5 cm and 6 cm. Then construct a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Sol. For correct construction of Δ 1

For construction of similar Δ 2

OR

Draw a circle of radius 2.5 cm. Take a point P at a distance of 8 cm from its centre. Construct a pair of tangents from the point P to the circle.

Sol. For draw the correct circle & exterior pt. 1

For construction of the pair of tangents 2

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28. If the n^{th} terms of two A.P.s 23, 25, 27, ... and 5, 8, 11, 14, ... are equal, then find the value of n.

Sol. nth term of first A.P = nth term of second A.P $\frac{1}{2}$

$23 + (n - 1)2 = 5 + (n - 1)3$ 1

$21 + 2n = 2 + 3n$ 1

$n = 19$ $\frac{1}{2}$

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29. In Figure-4, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle.

If OA = 7 cm, then find the area of the shaded region.

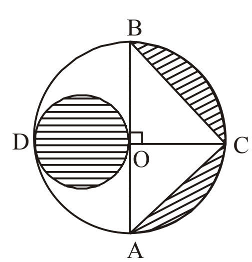

Fig. 4

Sol. Area of smaller circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} = 38.5 \text{ cm}^2$ 1

Area of Big semi-circle = $\frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$ $\frac{1}{2}$

Area of $\Delta ABC = \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$ $\frac{1}{2}$

Area of shaded portion = ar. of smaller circle + ar. of big semicircle – ar. of ΔABC

$= 38.5 + 77 - 49 = 66.5 \text{ cm}^2$ 1

OR

In Figure-5, ABCD is a square with side 7 cm. A circle is drawn circumscribing the square. Find the area of the shaded region.

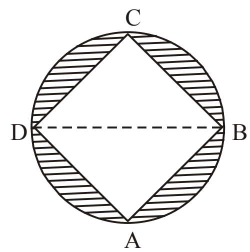

Fig. 5

Sol. Area of sqaure ABCD = $a^2 = 7^2 = 49 \text{ cm}^2$ $\frac{1}{2}$

Diagonal of square = $\sqrt{2}a = 7\sqrt{2}$ cm 1

$\therefore$ Radius of circle = $\frac{7\sqrt{2}}{2}$ cm $\frac{1}{2}$

Area of circle = $\frac{22}{7} \times \left(\frac{7\sqrt{2}}{2}\right)^2 = 77 \text{ cm}^2$ $\frac{1}{2}$

Area of shaded, portion = $77 - 49 = 28 \text{ cm}^2$ $\frac{1}{2}$

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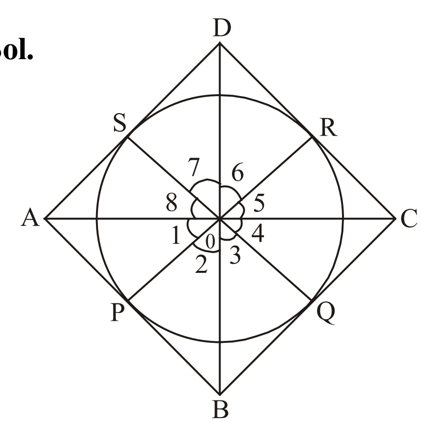
30. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol.



Correct figure $\frac{1}{2}$

$\Delta OAP \cong OAS$ [By SSS] 1

$\angle 1 = \angle 8$

Similarly $\angle 2 = \angle 3$

$\angle 4 = \angle 5 \quad \angle 6 = \angle 7$ $\frac{1}{2}$

Adding all angles

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$ $\frac{1}{2}$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$

$\angle AOB + \angle COD = 180°$ similarly $\angle BOC + \angle DOA = 180°$ $\frac{1}{2}$

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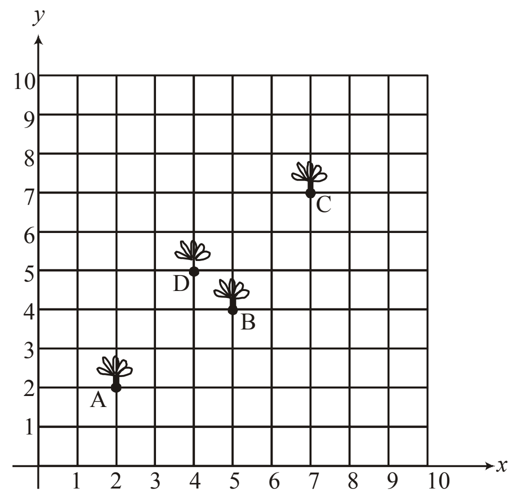
31. Krishna has an apple orchard which has a 10 m × 10 m sized kitchen garden attached to it. She divides it into a 10 × 10 grid and puts soil and manure into it. She grows a lemon plant at A, a coriander plant at B, an onion plant at C and a tomato plant at D. Her husband Ram praised her kitchen garden and points out that on joining A, B, C and D they may form a parallelogram. Look at the below figure carefully and answer the following questions:



(i) Write the coordinates of the points A, B, C and D, using the 10 × 10 grid as coordinate axes.

(ii) Find whether ABCD is a parallelogram or not.

Sol. (i) Coordinates are A(2, 2), B(5, 4), C(7, 7), D(4, 5) $4\times\frac{1}{2}=2$

(ii) AB = $\sqrt{(5-2)^2+(4-2)^2} = \sqrt{13}$

BC = $\sqrt{13}$

CD = $\sqrt{13}$

DA = $\sqrt{13}$ $\left[\begin{array}{l}\because AB = BC = CD = DA \\ \therefore \text{ABCD is a parallel gram}\end{array}\right]$ 1

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32. Prove that:

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A} = 2\sec A$$

Sol. LHS $= \dfrac{\cos^2 A+(1+\sin A)^2}{\cos A\,(1+\sin A)}$ 1

$= \dfrac{2(1+\sin A)}{\cos A(1+\sin A)}$ 1

$= \dfrac{2}{\cos A} = 2\sec A = \text{R.H.S}$ 1

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33. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ is rational

$\sqrt{3} = \dfrac{a}{b}$ (where a & b are +ve integers & co-prime, $b \neq 0$) $\frac{1}{2}$

$a^2 = 3b^2$ ...(i)

3 divides a^2

$\therefore$ 3 divides a also 1

Let a = 3c & put in (i)

$(3c)^2 = 3(b)^2$

$3c^2 = b^2$

$\Rightarrow$ 3 divides b^2

$\therefore$ 3 divides b also 1

$\therefore$ 3 divides a and b both

This contradicts our assumption

Therefore, $\sqrt{3}$ is irrational no. $\frac{1}{2}$

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34. The difference between two numbers is 26 and the larger number exceeds thrice of the smaller number by 4. Find the numbers.

Sol. Let larger No. = x

Let smaller No = y

$x - y = 26$ ...(i) 1

$x - 3y = 4$ ...(ii) 1

By solving (i) & (ii), we get

$\therefore$ $x = 37$ $\frac{1}{2}$

$y = 11$ $\frac{1}{2}$

OR

Solve for x and y:

$$\frac{2}{x}+\frac{3}{y} = 13 \text{ and } \frac{5}{x}-\frac{4}{y} = -2$$

Sol. Let $\frac{1}{x}=p$ & $\frac{1}{y}=q$

$2p + 3q = 13$ ...(i) $\frac{1}{2}$

$5p - 3q = -2$ ...(ii) $\frac{1}{2}$

By solving (i) & (ii), we get

$\therefore$ $p = 2, q = 3$ 1

$\therefore$ $\frac{1}{x}=2$ $\frac{1}{y}=3$

$\boxed{x=\frac{1}{2}}$ $\boxed{y=\frac{1}{3}}$ 1

SECTION D

Question numbers 35 to 40 carry 4 marks each.

35. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. Let smaller diameter tap takes x hours to fill the tank

Then, time taken by larger diameter tap to fill the tank = (x – 10) hr $\frac{1}{2}$

ATQ

$$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$$ $1\frac{1}{2}$

$8x^2 - 230x + 750 = 0$ $\frac{1}{2}$

$(8x - 30)(x - 25) = 0$ $\frac{1}{2}$

$x=\frac{15}{4}$ and $x=25$ $\frac{1}{2}$

Rejected $x = \frac{15}{4}$,

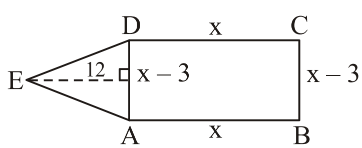
Hence, time taken by smaller diameter tap = 25 hrs

Time taken by larger diameter tap = 25 – 10 = 15 hrs $\frac{1}{2}$

OR

A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the Rectangular park and of altitude 12 m. Find the length and breadth of the park.

Sol.



Let length of rectangle = x Correct figure $\frac{1}{2}$

$\therefore$ Breadth = x – 3

ar. of reactangle = x(x – 3)

$= x^2 - 3x$ 1

Area of Isosceles ΔADE

$=\frac{1}{2}(x-3)\times 12$

$= 6x - 18$ $\frac{1}{2}$

ATQ

$x^2 - 3x = 6x - 18 + 4$ 1

$x^2 - 9x + 14 = 0$

$(x - 7)(x - 2) = 0$

$x = 7, x = 2$ Rejected $\frac{1}{2}$

$\therefore$ Length of rectangle = 7 cm

Breadth of rectangle = 4 cm $\frac{1}{2}$

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36. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.

Sol. Volume of sand in cylinderal bucket = Volume of sand in corres. heap $\frac{1}{2}$

$$\pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$$

$$\pi \times 18 \times 18 \times 32 = \frac{1}{3} \times \pi \times r_2^2 \times 24$$ 2

$r_2^2 = 1296$ $\frac{1}{2}$

$r_2 = 36$ cm

$l = \sqrt{(36)^2 + (24)^2}$

$= \sqrt{1872} = 12\sqrt{13}$ cm 1

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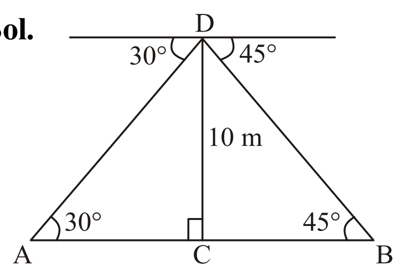
37. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 10 m from the banks, then find the width of the river. (Use $\sqrt{3}$ = 1.73)

Sol.



Correct figure 1

In ΔACD

$$\frac{1}{\sqrt{3}} = \frac{10}{AC}$$

$AC = 10\sqrt{3}$ m $1\frac{1}{2}$

In ΔBCD

$$1 = \frac{10}{BC}$$

BC = 10 m 1

Width of river (AB) = AC + BC = $10\sqrt{3} + 10 = 10(\sqrt{3} + 1)$ m $\frac{1}{2}$

38. Draw a 'less than' ogive for the following frequency distribution:

Classes:	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**
Frequency:	**7**	**14**	**13**	**12**	**20**	**11**	**15**	**8**

Sol.

getting the pts (10, 7), (20, 21)

(30, 34), (40, 46), (50, 66) 2

(60, 77), (70, 92), (80, 100)

Plotting and Joining the pts to get the correct ogive 2

39. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Sol. For correct given, To prove, Construction, Figure $4\times\frac{1}{2}=2$

For correct proof 2

OR

If Figure-6, in an equilateral triangle ABC, AD ⊥ BC, BE ⊥ AC and CF ⊥ AB.
Prove that $4(AD^2 + BE^2 + CF^2) = 9\,AB^2$.

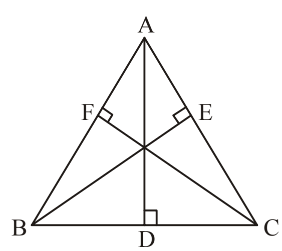


Figure 6

Sol. Proof

In ΔABD, $AD^2 = AB^2 - BD^2$ ...(i)

In ΔBCE $BE^2 = BC^2 - CE^2$ ...(ii)

In ΔACF $CF^2 = AC^2 - AF^2$ ...(iii)

$3\times\frac{1}{2}=1\frac{1}{2}$

$AD^2 + BE^2 + CF^2 = AB^2 + BC^2 + AC^2 - BD^2 - CE^2 - AF^2$ 1

$$= 3AB^2 - \left(\frac{BC}{2}\right)^2 - \left(\frac{AC}{2}\right)^2 - \left(\frac{AB}{2}\right)^2$$

$$= 3AB^2 - \frac{3}{4}AB^2$$

$$= \frac{9}{4}AB^2$$

$4(AD^2 + BE^2 + CF^2) = 9AB^2$ $1\frac{1}{2}$

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40. Find other zeroes of the polynomial
$p(x) = 3x^4 - 4x^3 - 10x^2 + 8x + 8$,
if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Sol. $\left(x - \sqrt{2}\right)$ & $\left(x + \sqrt{2}\right)$ are two factors

i.e. $x^2 - 2$ is a factor 1

$$\begin{array}{r|l}
 & 3x^2 - 4x - 4 \\
x^2 - 2 & 3x^4 - 4x^3 - 10x^2 + 8x + 8 \\
 & \underset{-}{3x^4} \qquad \underset{+}{- 6x^2} \\
\hline
 & - 4x^3 - 4x^2 + 8x + 8 \\
 & \underset{+}{- 4x^3} \qquad \underset{-}{+ 8x} \\
\hline
 & - 4x^2 + 8 \\
 & \underset{+}{- 4x^2} \underset{-}{+ 8} \\
\hline
 & 0
\end{array}$$ 2

$3x^2 - 4x - 4 = (3x + 2)(x - 2)$ $\frac{1}{2}$

$\therefore$ $-2/3$, 2 are other two zeroes. $\frac{1}{2}$

OR

Divide the polynomial g(x) = $x^3 - 3x^2 + x + 2$ by the polynomial $x^2 - 2x + 1$ and verify the division algorithm.

Sol.

$$
\begin{array}{r|l}
 & x - 1 \\
\hline
x^2 - 2x + 1 & \cancel{x^3} - 3x^2 + \cancel{x} + 2 \\
 & \cancel{x^3} - 2x^2 + \cancel{x} \\
 & - \quad + \quad - \\
\hline
 & -\cancel{x^2} + 2 \\
 & -\cancel{x^2} - 1 + 2x \\
 & + \quad + \quad - \\
\hline
 & -2x + 3 \\
\hline
\end{array}
$$
2

Verify,

P(x) = q(x) × g(x) + r(x)

$= (x - 1)(x^2 - 2x + 1) + (-2x + 3)$ 1

$= x^3 - 3x^2 + 3x - 1 - 2x + 3$

$= x^3 - 3x^2 + x + 2$ 1