

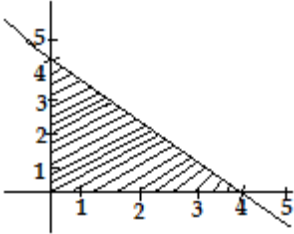
SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

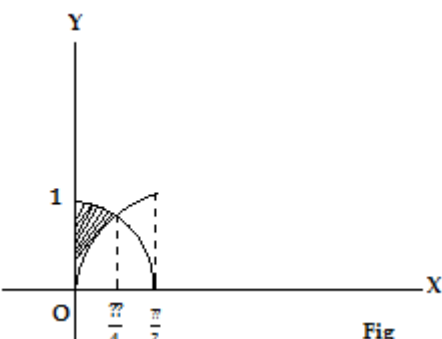
MATHEMATICS (041)

Marking Scheme

1.	$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\tan\frac{\pi}{3}\right) = -\frac{\pi}{3}$	1
2.	$ 3AB = 3^3 A B = 27 \times 2 \times 3 = 162$	1
3.	Distance of the point (p, q, r) from the x-axis $=$ Distance of the point (p, q, r) from the point (p,0,0) $= \sqrt{q^2 + r^2}$	1
4.	$\text{gof}(x) = g\{f(x)\} = g(3x^2 - 5) = \frac{3x^2 - 5}{(3x^2 - 5)^2 + 1} = \frac{3x^2 - 5}{9x^4 - 30x^2 + 26}$	1
5.	Equivalence relations could be the following: $\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ and (1) $\{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$ (1) So, only two equivalence relations.(Ans.)	2
6.	$AA' = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 \longrightarrow (1)$ because $l_i^2 + m_i^2 + n_i^2 = 1$, for each $i = 1, 2, 3 \longrightarrow 1/2$ $l_i l_j + m_i m_j + n_i n_j = 0$ ($i \neq j$) for each $i, j = 1, 2, 3 \longrightarrow 1/2$	2
7.	On differentiating $e^y (x + 1) = 1$ w.r.t. x, we get $e^y + (x + 1) e^y \frac{dy}{dx} = 0 \longrightarrow (1)$ $\Rightarrow e^y + \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -e^y \longrightarrow (1)$	2
8.	Here, $\left\{ \frac{d^2y}{dx^2} + (1 + x) \right\}^3 = -\frac{dy}{dx} \longrightarrow (1)$ Thus, order is 2 and degree is 3. So, the sum is 5 $\longrightarrow (1)$	2
9.	Here, $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{-6}$ is same as $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{6}$ Cartesian equation of the line is $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6} \longrightarrow (1)$ Vector equation of the line is $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k}) \longrightarrow (1)$	2

10.	The feasible region is a triangle with vertices $O(0,0)$, $A(4,0)$ and $B(0,4)$ $\left. \begin{aligned} Z_0 &= 3 \times 0 + 4 \times 0 = 0 \\ Z_A &= 3 \times 4 + 4 \times 0 = 12 \\ Z_B &= 3 \times 0 + 4 \times 4 = 16 \end{aligned} \right\} (1)$ Thus, maximum of Z is at $B(0,4)$ and the maximum value is 16 $\longrightarrow \frac{1}{2}$ 	2
11.	Sample space = $\{ B_1B_2, B_1G_2, G_1B_2, G_1G_2 \}$, B_1 and G_1 are the older boy and girl respectively. Let E_1 = both the children are boys; E_2 = one of the children is a boy ; E_3 = the older child is a boy Then, (i) $P(E_1/E_2) = P\left(\frac{E_1 \cap E_2}{E_2}\right) = \frac{1/4}{3/4} = \frac{1}{3} \longrightarrow (1)$ (ii) $P(E_1/E_3) = P\left(\frac{E_1 \cap E_3}{E_3}\right) = \frac{1/4}{2/4} = \frac{1}{2} \longrightarrow (1)$	2
12.	Here, $\text{Area}(A) = \frac{\sqrt{3}}{4} x^2$, where '$x$' is the side of the equilateral triangle $\longrightarrow \frac{1}{2}$ So, $\frac{dA}{dt} = \frac{\sqrt{3}}{2} x \frac{dx}{dt} \longrightarrow (1)$ $= \frac{\sqrt{3}}{2} (10) (2) = 10\sqrt{3} \text{ cm}^2/\text{sec} \longrightarrow \frac{1}{2}$	2
13.	As $A + B + C = \pi$, $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix} \longrightarrow (2)$ $= 0 \times \begin{vmatrix} 0 & \tan A \\ -\tan A & 0 \end{vmatrix} - \sin B \times \begin{vmatrix} -\sin B & \tan A \\ -\cos C & 0 \end{vmatrix} + \cos C \times \begin{vmatrix} -\sin B & 0 \\ -\cos C & -\tan A \end{vmatrix}$ $= 0 - \sin B \tan A \cos C + \cos C \sin B \tan A = 0 \text{ (Ans.)} \longrightarrow (2)$ OR	4

	Let $\Delta = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ Applying $C_1 \rightarrow C_1 + C_3$, we get $\Delta = (a+b+c) \begin{vmatrix} 1 & a-b & a \\ 1 & b-c & b \\ 1 & c-a & c \end{vmatrix} \longrightarrow (1)$ Applying $R_2 \rightarrow R_2 - R_1$, and $R_3 \rightarrow R_3 - R_1$, we get $\Delta = (a+b+c) \begin{vmatrix} 1 & a-b & a \\ 0 & 2b-a-c & b-a \\ 0 & 2a+b+c & c-a \end{vmatrix} \longrightarrow (1)$ Expanding Δ along first column, we have the result $\longrightarrow (2)$	4
14.	Since Rolle's theorem holds true, $f(1) = f(3)$ i.e., $(1)^3 - 6(1)^2 + a(1) + b = (3)^3 - 6(3)^2 + a(3) + b$ i.e., $a + b + 22 = 3a + b$ $\Rightarrow a = 11 \longrightarrow (2)$ Also, $f'(x) = 3x^2 - 12x + a$ or $3x^2 - 12x + 11$ As $f'(c) = 0$, we have $3\left(2 + \frac{1}{\sqrt{3}}\right)^2 - 12\left(2 + \frac{1}{\sqrt{3}}\right) + 11 = 0$ As it is independent of b, b is arbitrary. $\longrightarrow (2)$	4
15.	Here, $f'(x) = 3x^2 - 3x^{-4} = \frac{3(x^6 - 1)}{x^4} \longrightarrow (1)$ $= \frac{3(x^4 + x^2 + 1)}{x^4} (x+1)(x-1)$ Critical points are -1 and $1 \longrightarrow (1)$ $\Rightarrow f'(x) > 0$ if $x > 1$ or $x < -1$; and $f'(x) < 0$ if $-1 < x < 1$ $\left\{ \because \frac{3(x^4 + x^2 + 1)}{x^4} \text{ always +ive} \right\}$ Hence, $f(x)$ is strictly increasing for $x > 1 \longrightarrow (1)$ or $x < -1$; and strictly decreasing for $(-1, 0) \cup (0, 1) [1] \longrightarrow (1)$ OR Here, $\frac{dy}{dx} = 3x^2 - 11 \longrightarrow 1/2$ So, slope of the tangent is $3x^2 - 11$	4

	Slope of the given tangent line is 1. Thus, $3x^2 - 11 = 1 \longrightarrow (1)$ that gives $x = \pm 2$ When $x = 2, y = 2 - 11 = -9$ When $x = -2, y = -2 - 11 = -13$ Out of the two points $(2, -9)$ and $(-2, -13) \longrightarrow (2)$ only the point $(2, -9)$ lies on the curve Thus, the required point is $(2, -9) \longrightarrow \frac{1}{2}$	
16.	Here, $f(x) = x^2 + 3, a = 0, b = 2$ and $nh = b - a = 2 \longrightarrow (1)$ $\int_0^2 (x^2 + 1) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a + \overline{n-1}h)] \longrightarrow (1)$ $= \lim_{h \rightarrow 0} h [3 + 1^2h^2 + 3 + 2^2h^2 + 3 + \dots + (n-1)^2h^2 + 3]$ $= \lim_{h \rightarrow 0} h [3n + h^2 \{1^2 + 2^2 + 3^2 + \dots + (n-1)^2\}]$ $= \lim_{h \rightarrow 0} [3nh + h^3 \{ \frac{(n-1)n(2n-1)}{6} \}]$ $= \lim_{h \rightarrow 0} [3nh + \{ \frac{(nh-h)nh(2nh-h)}{6} \}] \longrightarrow (1)$ $= \lim_{h \rightarrow 0} [3 \times 2 + \{ \frac{(2-h)2(4-h)}{6} \}]$ $= 6 + \frac{16}{6}, \text{ i.e., } \frac{26}{3} \longrightarrow (1)$	4
17.	The rough sketch of the bounded region is shown on the right. $\longrightarrow (1)$ Required area $= \int_0^{\pi/4} \cos x dx - \int_0^{\pi/4} \sin x dx \longrightarrow (1)$ $= (\sin x + \cos x) \Big _0^{\pi/4} \longrightarrow (1)$ $= \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - \sin 0 - \cos 0$ $= \frac{2}{\sqrt{2}} - 1, \text{ i.e., } (\sqrt{2} - 1) \text{ sq units } \longrightarrow (1)$ 	4
18.	$y = ax + \frac{b}{a} \dots (1)$ gives $\frac{dy}{dx} = a \longrightarrow \left(1 \frac{1}{2}\right)$ Substituting this value of 'a' in (1), we get	4

	$y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \longrightarrow \left(1 \frac{1}{2}\right)$ Thus, $y = ax + \frac{b}{a}$ is a solution of the following differential equation $y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \longrightarrow 1$ OR Given differential equation can be written as $\frac{dy}{dx} = \frac{1+xy + \cos\left(\frac{y}{x}\right)}{x^2} = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right] \dots\dots(1)$ Let $F(x,y) = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right]$. Then $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \left[\frac{1 + \cos\left(\frac{\lambda y}{\lambda x}\right)}{(\lambda x)^2}\right]$ $= \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{\lambda^2 x^2}\right] \neq F(x,y)$ Hence, the given D.E. is not a homogeneous equation. $\longrightarrow (1)$ Putting $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ in (1), we get $v + x \frac{dv}{dx} = v + \frac{1 + \cos v}{x^2}$ $\Rightarrow \frac{dv}{1 + \cos v} = \frac{1}{x^3} dx$ $\Rightarrow \sec^2\left(\frac{v}{2}\right) dv = \frac{2}{x^3} dx \longrightarrow (1)$ Integrating both sides, we get $2 \tan \frac{v}{2} = -\frac{1}{x^2} + C \longrightarrow 1 \frac{1}{2}$ $\text{or } 2 \tan \frac{y}{2x} = -\frac{1}{x^2} + C \longrightarrow \frac{1}{2}$	4
19.	Since the vector $\vec{p}$, $\vec{q}$ and $\vec{r}$ are coplanar $\therefore [\vec{p}, \vec{q}, \vec{r}] = 0$ $[\vec{p} \quad \vec{q} \quad \vec{r}] = 0 \longrightarrow (1)$ i.e., $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$ $\begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{matrix} \longrightarrow (1)$ or $\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$	4

	$\Rightarrow a(b-1)(c-1) - 1(1-a)(c-1) - 1(1-a)(b-1) = 0$ i.e., $a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0 \longrightarrow (1)$ Dividing both the sides by $(1-a)(1-b)(1-c)$, we get $\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$ i.e., $-\left(1 - \frac{1}{1-a}\right) + \frac{1}{1-b} + \frac{1}{1-c} = 0$ i.e., $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1 \longrightarrow (1)$									
20.	We know that the equation of the plane having intercepts a, b and c on the three coordinate axes is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \longrightarrow (1)$ Here, the coordinates of A, B and C are (a,0,0), (0,b,0) and (0,0,c) respectively. The centroid of ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) \longrightarrow (1)$ Equating $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ to (α, β, γ) , we get $a = 3\alpha, b = 3\beta$ and $c = 3\gamma \longrightarrow (1)$ Thus, the equation of the plane is $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$ or $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3 \longrightarrow (1)$	4								
21.	Let the distance covered with speed of 25 km/h = x km and the distance covered with speed of 40 km/h = y km $\left(\frac{1}{2}\right)$ Total distance covered = z km The L.P.P. of the above problem, therefore, is $\longrightarrow (1)$ Maximize $z = x + y$ subject to constraints $\left\{ \begin{array}{l} 4x + 5y \leq 200 \\ \frac{x}{25} + \frac{y}{40} \leq 1 \\ x \geq 0, y \geq 0 \end{array} \right. \longrightarrow (1)$ Any one value $\longrightarrow \left(\frac{1}{2}\right)$	4								
22.	Here, <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>k</td><td>2k</td><td>3k</td></tr></table> (i) Since $P(0) + P(1) + P(2) = 1$, we have	X	0	1	2	P(X)	k	2k	3k	4
X	0	1	2							
P(X)	k	2k	3k							

	$k + 2k + 3k = 1$ $\text{i.e., } 6k = 1, \text{ or } k = \frac{1}{6} \longrightarrow (1)$ $\text{(ii) } P(X < 2) = P(0) + P(1) = k + 2k = 3k = \frac{1}{2}; \longrightarrow (1)$ $\text{(iii) } P(X \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 1 \longrightarrow (1)$ $\text{(iv) } P(X \geq 2) = P(2) = 3k = \frac{1}{2} \longrightarrow (1)$	
23.	Let the events be described as follows: E_1 : a coin having head on both sides is selected. E_2 : a fair coin is selected. A : head comes up in tossing a selected coin $P(E_1) = \frac{n}{2n+1}; P(E_2) = \frac{n+1}{2n+1}; P(A/E_1) = 1; P(A/E_2) = \frac{1}{2} \longrightarrow (2)$ It is given that $P(A) = \frac{31}{42}$. So, $P(E_1) P(A/E_1) + P(E_2) P(A/E_2) = \frac{31}{42}$ $\Rightarrow \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2} = \frac{31}{42} \longrightarrow (1)$ $\Rightarrow \frac{1}{2n+1} \left[n + \frac{n+1}{2} \right] = \frac{31}{42}$ $\Rightarrow 42(3n+1) = 62(2n+1)$ $\Rightarrow 2n = 20, \text{ or } n = 10 \longrightarrow (1)$	4
24.	$I = \int_0^{\pi} \frac{x}{1+\sin x} dx = \int_0^{\pi} \frac{\pi-x}{1+\sin(\pi-x)} dx \quad (1)$ $= \pi \int_0^{\pi} \frac{1}{1+\sin x} dx - \int_0^{\pi} \frac{x}{1+\sin x} dx$ $\Rightarrow 2I = \pi \int_0^{\pi} \frac{1}{1+\sin x} dx \quad (1)$ $\Rightarrow \frac{\pi}{2} \int_0^{\pi} \frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)} dx$ $\Rightarrow \frac{\pi}{2} \int_0^{\pi} \frac{1}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} dx$ $\Rightarrow \frac{\pi}{4} \int_0^{\pi} \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right) dx \quad (1)$ $\Rightarrow I = \frac{\pi}{4} \left[-2\tan\left[\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] \right]_0^{\pi} \quad (2)$ $\Rightarrow I = \frac{\pi}{4} [2 - (-2)] = \pi \quad (1)$ OR	6

	Let $I = \int \frac{\sin x}{\sin^3 x + \cos^3 x} dx = \int \frac{\tan x \sec^2 x}{\tan^3 x + 1} dx$ (½) On substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get (1) $I = \int \frac{t}{t^3 + 1} dt = \int \frac{t}{(t+1)(t^2 - t + 1)} dt$ (½) $= -\frac{1}{3} \int \frac{1}{t+1} dt + \frac{1}{3} \int \frac{t+1}{t^2 - t + 1} dt$ $= -\frac{1}{3} \log t+1 + \frac{1}{6} \int \frac{(2t-1)+3}{t^2 - t + 1} dt$ (1) $= -\frac{1}{3} \log t+1 + \frac{1}{6} \int \frac{2t-1}{t^2 - t + 1} dt + \frac{1}{2} \int \frac{1}{t^2 - t + 1} dt$ $= -\frac{1}{3} \log t+1 + \frac{1}{6} \log t^2 - t + 1 + \frac{1}{2} \int \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dt$ $= -\frac{1}{3} \log t+1 + \frac{1}{6} \log t^2 - t + 1 + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$ (2) $= -\frac{1}{3} \log \tan x + 1 + \frac{1}{6} \log \tan^2 x - \tan x + 1 + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + c$ (1)	6
25.	$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = -\tan^{-1} 7$ $\Rightarrow \tan^{-1}\left[\frac{\left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right)}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right] = -\tan^{-1} 7$, if $\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) < 1$(*) (2) $\Rightarrow \tan^{-1}\left[\frac{x(x+1) + (x-1)^2}{(x-1)x - (x+1)(x-1)}\right] = -\tan^{-1} 7$ $\Rightarrow \frac{(x^2+x) + (x^2+1-2x)}{(x^2-x) - (x^2-1)} = \tan[-\tan^{-1} 7]$ $\Rightarrow \frac{2x^2 - x + 1}{-x + 1} = -7$ (1) $\Rightarrow 2x^2 - 8x + 8 = 0$ $\Rightarrow (x - 2)^2 = 0$ $\Rightarrow x = 2$ (1) Let us now verify whether $x = 2$ satisfies the condition (*) For $x = 2$, $\left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) = 3 \times \frac{1}{2} = \frac{3}{2}$ which is not less than 1 Hence, this value does not satisfy the condition (*) (1) i.e., there is no solution to the given trigonometric equation. (1) OR Given * on Q, defined by $a*b = ab+1$ Let, $a \in Q, b \in Q$ then $ab \in Q$	6

	and $(ab+1) \in \mathbb{Q}$ $\Rightarrow a*b = ab+1$ is defined on $\mathbb{Q}$ $\therefore *$ is a binary operation on $\mathbb{Q}$ (1) Commutative: $a*b = ab+1$ $b*a = ba+1$ $= ab+1 \quad (\because ba = ab \text{ in } \mathbb{Q})$ $\Rightarrow a*b = b*a$ So $*$ is commutative on $\mathbb{Q}$ (1) Associative: $(a*b)*c = (ab+1)*c = (ab+1)c+1$ $= abc+c+1$ $a*(b*c) = a*(bc+1)$ $= a(bc+1)+1$ $= abc+a+1$ $\therefore (a*b)*c \neq a*(b*c)$ So $*$ is not associative on $\mathbb{Q}$ (1) Identity Element : Let $e \in \mathbb{Q}$ be the identity element, then for every $a \in \mathbb{Q}$ $a*e = a$ and $e*a = a$ $ae+1 = a$ and $ea+1 = a$ $\Rightarrow e = \frac{a-1}{a}$ and $e = \frac{a-1}{a}$ (1) e is not unique as it depend on 'a', hence identity element does not exist for $*$ (1) Inverse: since there is no identity element hence, there is no inverse. (1)	6
26.	The relation $A' = A^{-1}$ gives $A'A = A^{-1}A = I$ (1) Thus, $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \left(1 \frac{1}{2}\right)$ $\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-zx+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$ $\Rightarrow 2x^2 = 1; 6y^2 = 1 \text{ and } 3z^2 = 1$ $\Rightarrow x = \pm \frac{1}{\sqrt{2}}; y = \pm \frac{1}{\sqrt{6}}; z = \pm \frac{1}{\sqrt{3}} \quad \left(1 \frac{1}{2}\right)$ OR	6

	Here, $A = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0+0) + 1(9+2) + 2(0-0) = 11$ (1) $\Rightarrow A I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$(1) (1/2) $\text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$ (2) Now, $A(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1) and $(\text{adj } A)A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ (1) Thus, it is verified that $A(\text{adj } A) = (\text{adj } A)A = A I$ (1/2)	6
27.	Putting $x = \cos 2\theta$ in $\left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$, we get (1) $2 \tan^{-1} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ i.e., $2 \tan^{-1} \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} = 2 \tan^{-1}(\tan \theta) = 2\theta = \cos^{-1} x$ (2) Hence, $y = e^{\sin^2 x} \cos^{-1} x$ $\Rightarrow \log y = \sin^2 x + \log (\cos^{-1} x)$ $\Rightarrow \frac{1}{y} \times \frac{dy}{dx} = 2 \sin x \cos x + \frac{1}{\cos^{-1} x} \times \frac{-1}{\sqrt{1-x^2}} = \sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}}$ (2) $\Rightarrow \frac{dy}{dx} = e^{\sin^2 x} \cos^{-1} x \left[\sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}} \right]$ (1)	6
28.	Let (t^2, t) be any point on the curve $y^2 = x$. Its distance (S) from the line $x - y + 1 = 0$ is given by $1/2$ $S = \left \frac{t^2 - t - 1}{\sqrt{1+1}} \right \quad 1/2$ $= \frac{t^2 - t + 1}{\sqrt{2}} \quad \left\{ \because t^2 - t + 1 = \left(t - \frac{1}{2}\right)^2 + \frac{3}{4} > 0 \right\} \quad (1)$ $\Rightarrow \frac{dS}{dt} = \frac{1}{\sqrt{2}} (2t-1) \quad (1)$ and $\frac{d^2S}{dt^2} = \sqrt{2} > 0$ (1) Now, $\frac{dS}{dt} = 0 \Rightarrow \frac{1}{\sqrt{2}} (2t-1) = 0$,i.e., $t = \frac{1}{2}$ (1) Thus, S is minimum at $t = \frac{1}{2}$	6

So, the required shortest distance is $\frac{(\frac{1}{2})^2 - (\frac{1}{2}) + 1}{\sqrt{2}} = \frac{3}{4\sqrt{2}}$, or $\frac{3\sqrt{2}}{8}$ (1)

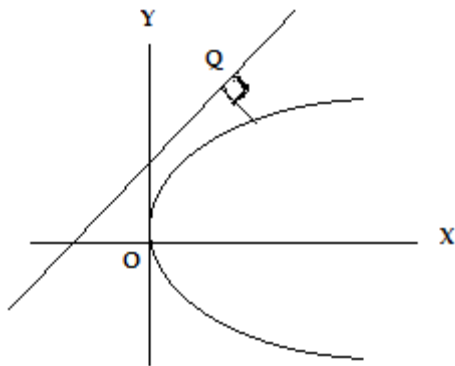


Fig. 1

29. 1) the line which are neither intersecting nor parallel. (1)

2) The given equations are

$$\vec{r} = 8\hat{i} - 9\hat{j} + 10\hat{k} + \mu(3\hat{i} - 16\hat{j} + 7\hat{k}) \dots\dots\dots(1) \quad (\frac{1}{2})$$

$$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}) \dots\dots\dots(2)$$

$$\text{Here, } \vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k}; \quad \vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}$$

$$\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}; \quad \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

$$\text{Now, } \vec{a}_2 - \vec{a}_1 = (15 - 8)\hat{i} + (29 + 9)\hat{j} + (5 - 10)\hat{k} = 7\hat{i} + 38\hat{j} - 5\hat{k} \quad (\frac{1}{2})$$

and

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k} \quad (1)$$

$$\Rightarrow (\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (24\hat{i} + 36\hat{j} + 72\hat{k}) \cdot (7\hat{i} + 38\hat{j} - 5\hat{k}) = 1176 \quad (1)$$

$$\text{Shortest distance} = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \quad (1)$$

$$= \left| \frac{1176}{\sqrt{24^2 + 36^2 + 72^2}} \right| = \frac{1176}{\sqrt{7056}} = \frac{1176}{84} = \frac{98}{7} \quad (1)$$