QUESTION PAPER CODE 65/5/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Order = 2 1

2. $\sin\theta = \left|\dfrac{(\hat{i}+\hat{j}+\hat{k})\cdot(3\hat{i}-\hat{j}+2\hat{k})}{\sqrt{1+1+1}\,\sqrt{9+1+4}}\right|$ $\frac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{4}{\sqrt{42}}\right)$ $\frac{1}{2}$

OR

Any point on $\dfrac{x+2}{1} = \dfrac{y-5}{3} = \dfrac{z+1}{5} = \lambda$ ...(1) $\frac{1}{2}$

is $(\lambda-2, 3\lambda+5, 5\lambda-1)$

Line (1) cuts yz plane at $\lambda - 2 = 0$ i.e., $\lambda = 2$

hence required point is (0, 11, 9) $\frac{1}{2}$

3. $|\text{adj } A| = |A|^{n-1}$ $\frac{1}{2}$

$|A| = 10$

$|\text{adj } A| = (10)^{2-1} = 10$ $\frac{1}{2}$

4. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\sec e^{2x}}}\cdot \sec e^{2x}\ \tan e^{2x} . 2e^{2x}$ 1

or $2\sqrt{\sec e^{2x}}\ \tan e^{2x}\ e^{2x}$

SECTION B

5. $n = 4,\ p = \dfrac{1}{4}, q = \dfrac{3}{4}$ $\frac{1}{2}$

P (atleast 3 are diamonds) = P (X = 3) + P(X = 4)

$= {}^4C_3\left(\dfrac{1}{4}\right)^3\left(\dfrac{3}{4}\right) + {}^4C_4\left(\dfrac{1}{4}\right)^4$ 1

$= \left(\frac{1}{4}\right)^4 [12+1] = \frac{13}{256}$ $\frac{1}{2}$

OR

Let E_1 : A coming on time.

E_2 : B coming on time.

$P(\bar{E}_1) = \frac{5}{7}, P(\bar{E}_2) = \frac{3}{7}$ $\frac{1}{2}$

P(only one on time)

$= P(E_1)\, P(\bar{E}_2) + P(\bar{E}_1)\, P(E_2)$

$= \frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7}$ 1

$= \frac{26}{49}$ $\frac{1}{2}$

6. Let $P(2, -1, 3), Q(3, -5, 1)$ and $R(-1, 11, 9)$ be three point.

$\overrightarrow{PQ} = \hat{i} - 4\hat{j} - 2\hat{k}$ $\frac{1}{2}$

$\overrightarrow{PR} = -3\hat{i} + 12\hat{j} + 6\hat{k} = -3(\hat{i} - 4\hat{j} - 2\hat{k})$ $\frac{1}{2}$

$\because \overrightarrow{PR} = -3\overrightarrow{PQ}$, since P is common.

Therefore the points P, Q and R are collinear. 1

OR

$(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$

LHS = $(\vec{a} \times \vec{b})^2$

$= (|\vec{a}||\vec{b}| \sin\theta\, \hat{n})^2 = |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta$ 1

$= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2\theta)$

$= |\vec{a}|^2 |\vec{b}|^2 - (|\vec{a}||\vec{b}| \cos\theta)^2$

$= \vec{a}^2\, \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$ 1

7. $\int \frac{x-1}{(x-2)(x-3)}\,dx = \int \left(\frac{-1}{x-2} + \frac{2}{x-3}\right) dx$ 1

$= -\log|x-2| + 2\log|x-3| + C$ 1

OR

$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$ Put $e^x = t$ so that $e^x\,dx = dt$ $\frac{1}{2}$

$\int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{3^2-(t+2)^2}}$ 1

$= \sin^{-1}\left(\frac{t+2}{3}\right) + C = \sin^{-1}\left(\frac{e^x+2}{3}\right) + C$ $\frac{1}{2}$

8. $P(B|A) = 0.4 \Rightarrow \frac{P(B \cap A)}{P(B)} = 0.4 \Rightarrow P(B \cap A) = 0.24$ $\frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.5 - 0.24 = 0.86$ 1

$P(A|B) = \frac{P(A \cap B)}{P(A)} = \frac{0.24}{0.5} = 0.48$ $\frac{1}{2}$

9. $\begin{bmatrix} 2x-3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$ 1

$\Rightarrow 2x - 3 = 7$ and $2y - 4 = 14 \Rightarrow x = 5,\ y = 9 \Rightarrow x - y = -4$ 1

10. $\int e^x(\cos x - \sin x)\,\text{cosec}^2 x\,dx = -\int e^x[\text{cosec}\,x - \text{cosec}\,x\cot x]\,dx$ 1

$= -e^x\,\text{cosec}\,x + c$ 1

11. $y^2 = a(b^2 - x^2)$

$2yy' = -2ax$ $\frac{1}{2}$

$\Rightarrow \frac{yy'}{x} = -a$ $\frac{1}{2}$

$\Rightarrow \frac{(y' \cdot y' + yy'')\,x - yy'.1}{x^2} = 0$

$\Rightarrow \; x(y')^2 + xyy'' - yy' = 0 \text{ or } x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} - y\frac{dy}{dx} = 0$ 1

12. $a \in N, b \in N \Rightarrow a^b \in N \Rightarrow a * b \in N \Rightarrow *$ is binary operation. 1

In general $a^b \neq b^a$, for $a, b \in N \Rightarrow *$ is not commutative. 1

SECTION C

13. $(1+e^{2x})dy + (1+y^2)e^x\, dx = 0$

$\Rightarrow \int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}}\, dx$ 1

$\Rightarrow \tan^{-1} y = -\int \frac{e^x}{1+e^{2x}}\, dx$ $\frac{1}{2}$

Put $e^x = t$, so that $e^x\, dx = dt$ $\frac{1}{2}$

$\tan^{-1} y = -\int \frac{dt}{1+t^2} \Rightarrow \tan^{-1} y = -\tan^{-1}(e^x) + C$ ...(i) 1

Substituting y = 1, when x = 0 in equation (i)

$\tan^{-1}(1) = -\tan^{-1}(1) + C \Rightarrow C = \frac{\pi}{2}$ $\frac{1}{2}$

Substituting $C = \frac{\pi}{2}$ in equation (i) $\Rightarrow \tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$ $\frac{1}{2}$

OR

$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin\left(\frac{y}{x}\right)}$ ...(i) $\frac{1}{2}$

Put $\frac{y}{x} = v$ i.e., $y = vx$ in (i) so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$v + x\frac{dv}{dx} = v - \frac{1}{\sin v}$

$\int \sin v \, dv = \int -\frac{1}{x} dx$ $\frac{1}{2}$

$\Rightarrow -\cos v = -\log|x| + C \Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + C$ ...(ii) 1

Substituting $y = \frac{\pi}{2}$ when x = 1 in (ii)

$\cos\left(\frac{\pi}{2}\right) = \log 1 + C \Rightarrow C = 0$ $\frac{1}{2}$

Required solution is $\cos\left(\frac{y}{x}\right) = \log|x|$ $\frac{1}{2}$

14. (i) **Reflexive:** $\forall a \in A, |a - a| = 0$ which is even

$\Rightarrow (a, a) \in R$, hence R is reflexive 1

(ii) **Symmetric:** Let $(a, b) \in R \Rightarrow |a - b|$ is even

$\Rightarrow |-(b - a)|$ is even $\Rightarrow |b - a|$ is even

so, $(b, a) \in R$

hence R is symmetric. 1

(iii) **Transitive:** Let $(a, b), (b, c) \in R$

so, $|a - b|$ is even and $|b - c|$ is even

$\Rightarrow a - b = 2\lambda, b - c = 2\mu$ where $\lambda, \mu \in Z$

Now, $a - c = (a - b) + (b - c) = 2(\lambda + \mu)$

$\Rightarrow |a - c|$ is even, so $(a, c) \in R$

hence R is transitive. $1\frac{1}{2}$

Since R is reflexive, symmetric and transitive therefore its an equivalence relation $\frac{1}{2}$

OR

Let for $x_1, x_2 \in A$, $f(x_1) = f(x_2)$ $\frac{1}{2}$

$$\frac{4x_1 + 3}{6x_1 - 4} = \frac{4x_2 + 3}{6x_2 - 4}$$

$\Rightarrow (4x_1+3)(6x_2-4) = (6x_1-4)(4x_2+3)$

$\Rightarrow 34x_1 = 34x_2 \Rightarrow x_1 = x_2$, hence f is one-one. 1

For any $y \in A$ such that $y = \dfrac{4x+3}{6x-4}$ there exists x such that

$6xy - 4y = 4x + 3 \Rightarrow (6y-4)x = 4y+3$

$\Rightarrow x = \dfrac{4y+3}{6y-4}, y \in A, x = \dfrac{4y+3}{6y-4} \in A$ 1

$\Rightarrow$ f is onto. 1

Since f is one-one and onto, therefore f^{-1}exists in A

and $f^{-1}(y) = \dfrac{4y+3}{6y-4}$ or $f^{-1}(x) = \dfrac{4x+3}{6x-4}$ $\frac{1}{2}$

15. $f(x) = \cos\left(2x+\dfrac{\pi}{4}\right) \Rightarrow f'(x) = -2\sin\left(2x+\dfrac{\pi}{4}\right)$ $1\frac{1}{2}$

As $\dfrac{3\pi}{8} < x < \dfrac{5\pi}{8} \Rightarrow \dfrac{3\pi}{4} < 2x < \dfrac{5\pi}{4}$

$\Rightarrow \pi < 2x + \dfrac{\pi}{4} < \dfrac{3\pi}{2}$ $1\frac{1}{2}$

$\Rightarrow \sin\left(2x+\dfrac{\pi}{4}\right) < 0 \Rightarrow f'(x) > 0$

$\therefore$ f(x) is increasing in $\left(\dfrac{3\pi}{8}, \dfrac{5\pi}{8}\right)$ 1

16. A(x, 5, –1), B(3, 2, 1), C(4, 5, 5), D(4, 2, –2)

$$\left.\begin{array}{l}\overrightarrow{BA} = (x-3)\hat{i} + 3\hat{j} - 2\hat{k} \\ \overrightarrow{BC} = 1\hat{i} + 3\hat{j} + 4\hat{k} \\ \overrightarrow{BD} = 1\hat{i} + 0\hat{j} - 3\hat{k}\end{array}\right\}$$ $1\frac{1}{2}$

$$\begin{vmatrix} x-3 & 3 & -2 \\ 1 & 3 & 4 \\ 1 & 0 & -3 \end{vmatrix} = 0$$ 1

i.e., (x − 3) (−9) −3(−7) −2(−3) = 0 1

$\Rightarrow$ x = 6 $\frac{1}{2}$

17. LHS = $\sin^{-1}\frac{4}{5}+\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{63}{65}$

$= \tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{12}+\cos^{-1}\frac{63}{65}$ 1

$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}\right)+\cos^{-1}\frac{63}{65}$ 1

$= \tan^{-1}\left(\frac{63}{16}\right)+\cot^{-1}\left(\frac{63}{16}\right)$ 1

$= \frac{\pi}{2} = \text{RHS}$ 1

18. $x^p y^q = (x+y)^{p+q} \Rightarrow p\log x + q\log y = (p+q)\log(x+y)$ $\frac{1}{2}$

Differentaiting w.r.t x, $\frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$ 1

$\Rightarrow \left(\frac{q}{y}-\frac{p+q}{x+y}\right)\frac{dy}{dx}=\frac{p+q}{x+y}-\frac{p}{x}$

$\Rightarrow \left[\frac{qx-py}{y(x+y)}\right]\frac{dy}{dx}=\frac{qx-py}{x(x+y)} \Rightarrow \frac{dy}{dx}=\frac{y}{x}$ 1

Differentiating again w.r.t x

$\frac{d^2y}{dx^2}=\frac{x\frac{dy}{dx}-y}{x^2}$ 1

$\Rightarrow \frac{d^2y}{dx^2}=\frac{x\left(\frac{y}{x}\right)-y}{x^2} \Rightarrow \frac{d^2y}{dx^2}=0$ $\frac{1}{2}$

19. $\int (\sin x \sin 2x \sin 3x)\, dx = \frac{1}{2}\int (2\sin x \sin 2x) \sin 3x\, dx$ $\frac{1}{2}$

$= \frac{1}{2}\int [\cos(x) - \cos(3x)] \sin 3x\, dx$ 1

$= \frac{1}{4}\int 2\cos x \sin 3x\, dx - \frac{1}{4}\int 2\cos 3x \sin 3x\, dx$

$= \frac{1}{4}\int (\sin 4x + \sin 2x) dx - \frac{1}{4}\int \sin 6x\, dx$ 1

$= -\frac{1}{4}\left(\frac{\cos 4x}{4} + \frac{\cos 2x}{2}\right) + \frac{1}{24}\cos 6x + C$ $1\frac{1}{2}$

20. Let $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$, Put $x = \tan\theta$ $\frac{1}{2}$

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right) \Rightarrow y = \tan^{-1}(\tan 3\theta) = 3\theta$

$y = 3\tan^{-1} x \Rightarrow \frac{dy}{dx} = \frac{3}{1 + x^2}$ ...(i) 1

Let $z = \tan^{-1}\left(\frac{x}{\sqrt{1 - x^2}}\right)$, put $x = \sin\phi$ $\frac{1}{2}$

$z = \tan^{-1}\left(\frac{\sin\phi}{\sqrt{1 - \sin^2\phi}}\right) \Rightarrow z = \tan^{-1}(\tan\phi) = \phi$

$z = \phi = \sin^{-1} x \Rightarrow \frac{dz}{dx} = \frac{1}{\sqrt{1 - x^2}}$ ...(ii) 1

Using (i) & (ii), $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{3\sqrt{1 - x^2}}{1 + x^2}$ 1

OR

$\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, put $x = \sin\theta$, $y = \sin\phi$ 1

$$\sqrt{1-\sin^2\theta}+\sqrt{1-\sin^2\phi}=a(\sin\theta-\sin\phi)$$

$\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow \; 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)=2a\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$ 1

$\Rightarrow \; \tan\left(\frac{\theta-\phi}{2}\right)=\frac{1}{a}$

$\Rightarrow \; \frac{\theta-\phi}{2}=\tan^{-1}\left(\frac{1}{a}\right)\Rightarrow \sin^{-1}x-\sin^{-1}y=2\tan^{-1}\left(\frac{1}{a}\right)$ 1

Differentiating both sides w.r.t x

$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$ $\frac{1}{2}$

$\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$ or $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ $\frac{1}{2}$

21. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$ Taking a, b, c common from C_1, C_2, C_3 respectively 1

$= abc\begin{vmatrix} a & c & 0 \\ a+b & b & -2b \\ b & b+c & -2b \end{vmatrix}$ $C_3 \rightarrow C_3 - (C_1 + C_2)$ 1

$= abc\,.\,2b\begin{vmatrix} a & c & 0 \\ a+b & b & -1 \\ b & b+c & -1 \end{vmatrix}$

$= 2ab^2c\begin{vmatrix} a & c & 0 \\ a+b & b & -1 \\ -a & c & 0 \end{vmatrix}$ $R_3 \rightarrow R_3 - R_2$ 1

Expanding along C_3

$= 2ab^2c \ . \ 2ac = 4a^2b^2c^2$ 1

22. $I = \int_1^2 |x^3 - x| dx$

$$I = \int_{-1}^{0} (x^3 - x)dx - \int_0^1 (x^3 - x)dx + \int_1^2 (x^3 - x)dx$$ 2

$$I = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2$$ 1

$$I = -\left(-\frac{1}{4}\right) - \left(-\frac{1}{4}\right) + \frac{9}{4} = \frac{11}{4}$$ 1

23. Equation of planes passing through the intersection of given planes

$$[\vec{r}.(2\hat{i} + 6\hat{j}) + 12] + \lambda[\vec{r}.(3\hat{i} - \hat{j} + 4\hat{k})] = 0$$ 1

$$\Rightarrow \ \vec{r}.[(2 + 3\lambda)\hat{i} + (6 - \lambda)\hat{j} + 4\lambda\hat{k}] + 12 = 0$$ $\frac{1}{2}$

Now

$$1 = \frac{12}{\sqrt{(2 + 3\lambda)^2 + (6 - \lambda)^2 + (4\lambda)^2}}$$ $\frac{1}{2}$

$$1 = \frac{144}{26\lambda^2 + 40}$$

$$\Rightarrow \ 26\lambda^2 + 40 = 144 \Rightarrow \lambda = \pm 2$$ 1

Equation of plane is

$$\vec{r}.(8\hat{i} + 4\hat{j} + 8\hat{k}) + 12 = 0 \ \text{ or } \ \vec{r}.(2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0$$ $\frac{1}{2}$

and $\vec{r}.(-4\hat{i} + 8\hat{j} - 8\hat{k}) + 12 = 0$ or $\vec{r}.(-\hat{i} + 2\hat{j} - 2\hat{k}) + 3 = 0$ $\frac{1}{2}$

SECTION D

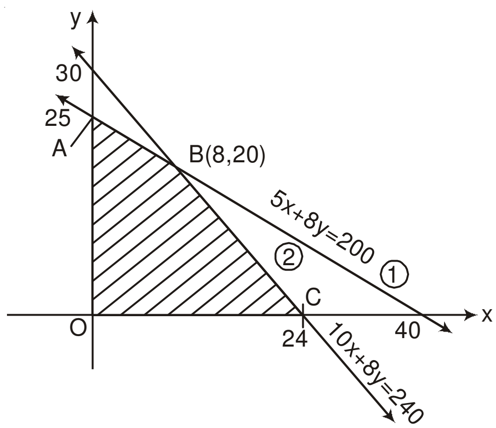
24. Let numbers of souvenirs of type A be x and number of souvenirs of type B be y

$\therefore$ L.P.P is

Maximize P = 100x + 120y $\frac{1}{2}$

Subject to constraints

$$\left.\begin{aligned} 5x + 8y &\le 200 \quad ...(1) \\ 10x + 8y &\le 240 \quad ...(2) \\ x, y &\ge 0 \end{aligned}\right\}$$ $2\frac{1}{2}$



2

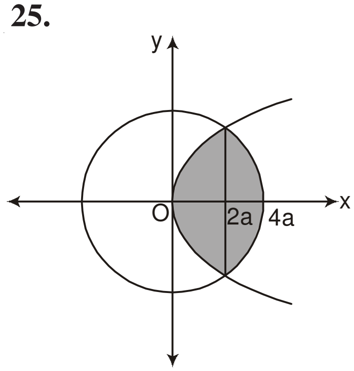
Values at corner points

Points	P
A(0, 25)	3000
B(8, 20)	3200 (Max)
C(24, 0)	2400

1

So, 8 type A Souvenirs and 20 type B Souvenirs should be made to maximize profit.

25. Correct figure 1



Solving $x^2 + y^2 = 16a^2$ ...(1)

and $y^2 = 6ax$ ...(2) we get

x = 2a (as –8a is not possible) 1

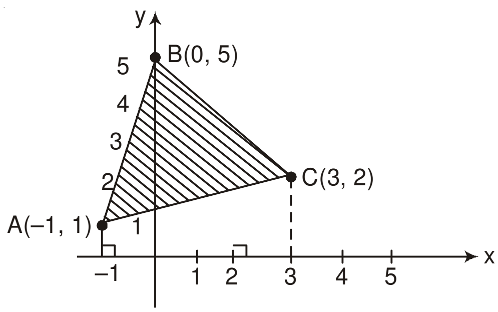
Required Area = $2\left[\int_0^{2a} \sqrt{6ax}\, dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2}\, dx\right]$ 2

$= 2\left[\sqrt{6a}\left(\frac{2}{3}x^{3/2}\right)\right]_0^{2a} + 2\left[\frac{x}{2}\sqrt{(4a)^2 - x^2} + 8a^2 \sin^{-1}\left(\frac{x}{4a}\right)\right]_{2a}^{4a}$ 1

$$= \frac{8}{3}\sqrt{12}a^2 + \frac{16}{3}\pi a^2 - 4\sqrt{3}a^2$$

$$= \frac{4}{3}a^2(4\pi + \sqrt{3})$$ 1

OR



Correct figure 1

$4x - y + 5 = 0$ ...(1)

$x + y - 5 = 0$ ...(2)

$x - 4y + 5 = 0$ ...(3)

Coordinates of A(–1, 1), B(0, 5) and C(3, 2) $1\frac{1}{2}$

Required Area = $\int_{-1}^{0}(4x+5)dx + \int_{0}^{3}(5-x)dx - \frac{1}{4}\int_{-1}^{3}(x+5)dx$ 2

$$= \left[2x^2 + 5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^{3}$$ 1

$$= \frac{15}{2}$$ $\frac{1}{2}$

26. Required equation of the line is

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + \hat{k})$ 2

Let $\vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}, \vec{a}_2 = \hat{i} + \hat{j}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

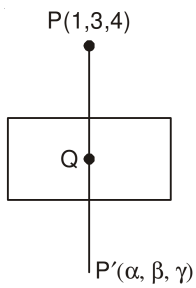
The required distance = $\left|\frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|}\right|$

$$= \frac{|(\hat{i} - \hat{k}) \times (2\hat{i} - \hat{j} + \hat{k})|}{|2\hat{i} - \hat{j} + \hat{k}|}$$ 1

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix} = -\hat{i} - 3\hat{j} - \hat{k}$$ 2

Required distance = $\dfrac{\sqrt{1+9+1}}{\sqrt{4+1+1}} = \dfrac{\sqrt{11}}{\sqrt{6}}$ or $\dfrac{\sqrt{66}}{6}$ 1

OR



Correct figure $\frac{1}{2}$

Equation of line PQ is $\dfrac{x-1}{2} = \dfrac{y-3}{-1} = \dfrac{z-4}{1} = \lambda$ 1

The coordinates of Q are $(2\lambda + 1, -\lambda + 3, \lambda + 4)$ $\frac{1}{2}$

$\because$ Q lies on plane $2x - y + z + 3 = 0$

$\therefore$ $2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$ 1

$\Rightarrow 6\lambda + 6 = 0$ i.e., $\lambda = -1$ $\frac{1}{2}$

The coordinates of Q are (–1, 4, 3) $\frac{1}{2}$

$PQ = \sqrt{(-1-1)^2 + (4-3)^2 + (3-4)^2} = \sqrt{6}$ 1

Let $P'(\alpha, \beta, \gamma)$ be the image of P.

then $\dfrac{\alpha+1}{2} = -1, \ \dfrac{\beta+3}{2} = 4, \dfrac{\gamma+4}{2} = 3$ $\frac{1}{2}$

$\Rightarrow \alpha = -3, \ \beta = 5, \ \gamma = 2$

$\therefore$ the image P′ is (–3, 5, 2) $\frac{1}{2}$

27. Let A = $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Then A = IA

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \qquad (R_3 \to R_3 - 4R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -13 & 9 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \qquad (R_1 \to R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A \qquad (R_1 \to R_2 - 3R_2)$$

4

$$\therefore\ A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$$ 1

OR

The given system of equations is

AX = B,

where $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$|A| = 1200 \neq 0$ 1

$\Rightarrow A^{-1}$ exists.

$X = A^{-1}B$ 1

$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ 2

$$A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$ $\frac{1}{2}$

$$X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$ 1

$$\therefore\ x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$$ $\frac{1}{2}$

28. Let x be the radius of circle and y be the side of square

$2\pi x + 4y = k$ 1

$A = \pi x^2 + y^2$

$$A = \pi x^2 + \left(\frac{k - 2\pi x}{4}\right)^2 = \frac{16\pi x^2 + k^2 + 4\pi^2 x^2 - 4\pi kx}{16}$$ 1

$$\frac{dA}{dx} = \frac{1}{16}(32\pi x + 8\pi^2 x - 4\pi k)$$ 1

$$\frac{dA}{dx} = 0 \Rightarrow 32\pi x + 8\pi^2 x - 4\pi k = 0$$

$$\Rightarrow\ x = \frac{k}{8 + 2\pi}$$ 1

$$\left.\frac{d^2A}{dx^2}\right]_{x=\frac{k}{8+2\pi}} = \frac{1}{16}[32\pi + 8\pi^2] > 0 \Rightarrow \text{Sum of areas is minimum}$$ 1

$$2\pi\left(\frac{k}{8+2\pi}\right)+4y=k \Rightarrow y=\frac{k}{4+\pi} \Rightarrow y=2x$$ 1

29. $\left.\begin{array}{l} E_1 : \text{ball drawn from first bag} \\ E_2 : \text{ball drawn from second bag} \\ A : \text{both drawn balls are red} \end{array}\right\}$ 1

$$P(E_1)=P(E_2)=\frac{1}{2}$$ 1

$$P(A\,|\,E_1)=\frac{5}{8}\times\frac{4}{7}=\frac{20}{56}$$ 1

$$P(A\,|\,E_2)=\frac{2}{8}\times\frac{1}{7}=\frac{2}{56}$$ 1

$$P(E_1\,|\,A)=\frac{\frac{1}{2}.\frac{20}{56}}{\frac{1}{2}.\frac{20}{56}+\frac{1}{2}.\frac{2}{56}}$$ $1\frac{1}{2}$

$$=\frac{\frac{20}{112}}{\frac{22}{112}}=\frac{10}{11}$$ $\frac{1}{2}$