QUESTION PAPER CODE 65/1/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $3A = \begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ $\frac{1}{2}+\frac{1}{2}$

2. Order = 2, degree = 2 $\frac{1}{2}+\frac{1}{2}$

3. (fof) (x) = f(x + 1) = x + 2 $\frac{1}{2}$

$\frac{d}{dx}(fof)(x) = 1$ $\frac{1}{2}$

4. d.c.'s = <cos 90°, cos 135°, cos 45°> $\frac{1}{2}$

$= < 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k})+\lambda(2\hat{i}+2\hat{j}-3\hat{k})$ 1

SECTION B

5. $I = \int \sin x \cdot \log(\cos x)\, dx$

$\cos x = t \Rightarrow I = -\int \log t \cdot 1 dt$ 1

$= -\left[t.\log t - \int \frac{1}{t}.t dt \right]$ $\frac{1}{2}$

$= t(1 - \log t) + C = \cos x(1 - \log(\cos x)) + C$ $\frac{1}{2}$

6. Let $f(x) = (1 - x^2) . \sin x \cos^2 x$

as $f(-x) = -f(x) \Rightarrow$ f is odd function. 1

$\therefore\ I = 0$ 1

OR

$$I = \int_{-1}^{2} \frac{|x|}{x} dx = \int_{-1}^{0} -1 dx + \int_{0}^{2} 1 dx$$ 1

$= -1 + 2 = 1$ 1

7. As $a, b \in R \Rightarrow ab \in R \Rightarrow ab + 1 \in R \Rightarrow a*b \in R \Rightarrow *$ is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general $(a*b)*c \neq a*(b*c) \Rightarrow *$ is not associative. 1

8. $$2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ 1

$$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$ 1

9. $A = \{2, 4, 6\}$, $B = \{1, 2, 3\}$, $A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

10. $y' = be^{2x} + 2y \Rightarrow b = \frac{y' - 2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$$\frac{e^{2x}.(y'' - 2y') - (y' - 2y).2x^{2x}}{(e^{2x})^2} = 0$$ 1

$\Rightarrow y'' - 4y' + 4y = 0$ or $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ $\frac{1}{2}$

11. Let X: getting an odd number

$p = \frac{1}{2},\ q = \frac{1}{2},\ n = 6$ $\frac{1}{2}$

(i) $P(X = 5) = {}^6C_5 \left(\frac{1}{2}\right)^6 = \frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X \le 5) = 1 - P(X = 6) = 1 - \frac{1}{64} = \frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow\ k = \frac{1}{6}$ 1

12. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow\ 1 + |\vec{a} - \vec{b}|^2 = 2(1 + 1)$

$\Rightarrow\ |\hat{a} - \hat{b}|^2 = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}$ 1

OR

$[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$ 1

$= -30$ 1

SECTION C

13. LHS $= \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

$R_1 \to R_1 + R_3$ and taking $a + b + c$ common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$ 1

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ a-2b+c & b-2c+a & c-a \\ b-a & c-b & a+b \end{vmatrix}$$ 1

$= (a + b + c)\ [(a - 2b + c)\ (c - b) - (b - 2c + a)\ (b - a)]$ 1

$= (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc = \text{RHS}.$ 1

14. $\tan^{-1}\left(\dfrac{4x+6x}{1-(4x)(6x)}\right) = \dfrac{\pi}{4}$ 1

$\Rightarrow \dfrac{10x}{1-24x^2} = 1 \Rightarrow 24x^2 + 10x - 1 = 0$ $1\frac{1}{2}$

$\Rightarrow x = \dfrac{1}{12}$ or $-\dfrac{1}{2}$ 1

as $x = -\dfrac{1}{2}$ does not satisfy the given equation, so $x = \dfrac{1}{12}$ $\frac{1}{2}$

15. Clearly $a \le a\ \forall\ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive. $1\frac{1}{2}$

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2,1) \notin R$

$\Rightarrow$ R is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ $(\because x_1, x_2 \in N)$ $1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \dfrac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \dfrac{\sqrt{4x-3}-1}{2}$ 1

16. Let the point of contact be $P(x_1, y_1)$

$\dfrac{dy}{dx} = \dfrac{3}{2\sqrt{3x-2}}$ (slope of tangent)

$\Rightarrow m_1 = \left.\dfrac{dy}{dx}\right]_{(x_1, y_1)} = \dfrac{3}{2\sqrt{3x_1 - 2}}$ 1

also, slope of given line = 2 = m_2

$m_1 = m_2 \Rightarrow x_1 = \dfrac{41}{48}$ 1

when $x_1 = \dfrac{41}{48}, y_1 = \sqrt{\dfrac{41}{16} - 2} = \dfrac{3}{4}$ $\therefore P\left(\dfrac{41}{48}, \dfrac{3}{4}\right)$ $\frac{1}{2}$

Equation of tangent is: $y-\frac{3}{4}=2\left(x-\frac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y-\frac{3}{4}=\frac{-1}{2}\left(x-\frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

17. $\log(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)=2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right)$$ 2

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right)=\frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right)$$ 1

$$\Rightarrow (x+y)=(x-y)\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{x+y}{x-y}$$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx}-\frac{dv}{dx}=0 \quad ...(1)$$ 1

Now, log u = y.logx

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx}=y\cdot\frac{1}{x}+\log x\cdot\frac{dy}{dx}\Rightarrow\frac{du}{dx}=x^y\left(\frac{y}{x}+\log x\cdot\frac{dy}{dx}\right) \quad ...(2)$$ 1

Again, $\log v = x \cdot \log y$

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx}=x\cdot\frac{1}{y}\cdot\frac{dy}{dx}+\log y\cdot 1\Rightarrow\frac{dv}{dx}=y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right) \quad ...(3)$$ 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x}+\log x\frac{dy}{dx}\right)-y^x\left(\frac{x}{y}\frac{dy}{dx}+\log y\right)=0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx}=\frac{y^x\cdot\log y-x^{y-1}\cdot y}{x^y\cdot\log x-y^{x-1}\cdot x}$$ $\frac{1}{2}$

18. $y=(\sin^{-1}x)^2$

$$\Rightarrow y'=2\cdot\sin^{-1}x\cdot\frac{1}{\sqrt{1-x^2}}$$ 1

$$\Rightarrow \sqrt{1-x^2}\cdot y'=2\sin^{-1}x$$

$$\Rightarrow \sqrt{1-x^2}\cdot y''+y'\cdot\frac{1}{2\sqrt{1-x^2}}(-2x)=\frac{2}{\sqrt{1-x^2}}$$ 2

$$\Rightarrow (1-x^2)\cdot y''-xy'=2 \text{ or } (1-x^2)\cdot\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0.$$ 1

19. Let $I=\int_0^a f(a-x)\,dx$

Put $a-x=t \Rightarrow -dx=dt$ $\frac{1}{2}$

$$I=-\int_a^0 f(t)\,dt=\int_0^a f(t)\,dt=\int_0^a f(x)\,dx$$ $\frac{1}{2}$

II part.

$$I=\int_0^\pi\frac{x\sin x}{1+\cos^2x}dx$$

$$\Rightarrow \quad I=\int_0^\pi\frac{(\pi-x)\cdot\sin x}{1+\cos^2x}dx$$

$$\Rightarrow \quad 2I=\int_0^\pi\frac{\pi\cdot\sin x}{1+\cos^2x}dx$$ $1\frac{1}{2}$

Put cos x = t ⇒ –sinx dx = dt

$$\Rightarrow \; I = -\frac{\pi}{2}\cdot\int_{1}^{-1}\frac{dt}{1+t^2} = \frac{\pi}{2}\times 2\times\int_{0}^{1}\frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$ $1\frac{1}{2}$

20. $I = \int \frac{\cos x}{(1+\sin x)\,(2+\sin x)}\,dx.$ Put sin x = t $\frac{1}{2}$

$$= \int \frac{dt}{(1+t)\,(2+t)} = \int\left(\frac{1}{1+t} - \frac{1}{2+t}\right)dt$$ 2

$$= \log\left|\frac{1+t}{2+t}\right| + c = \log\left|\frac{1+\sin x}{2+\sin x}\right| + c$$ $1+\frac{1}{2}$

21. I.F. $= e^{-\int \frac{2x}{1+x^2}dx} = \frac{1}{1+x^2}$ 1

Solution is given by,

$$y\cdot\left(\frac{1}{1+x^2}\right) = \int\frac{x^2+2}{1+x^2}\,dx$$ $1\frac{1}{2}$

$$y\cdot\frac{1}{1+x^2} = \int\left(1+\frac{1}{1+x^2}\right)dx = x + \tan^{-1} x + c$$ $1\frac{1}{2}$

or $y = (1 + x^2)\,(x + \tan^{-1}x + c)$

OR

Given equation can be written as

$$\int\frac{dy}{2e^{-y}-1} = \int\frac{dx}{x+1}$$

$$\Rightarrow \; \int\frac{e^y}{2-e^y}\,dy = \int\frac{dx}{x+1}$$ 1

$$\Rightarrow \; -\log|2-e^y| + \log c = \log|x+1|$$ $1\frac{1}{2}$

$\Rightarrow (2 - e^y)\,(x + 1) = c$

When x = 0, y = 0 $\Rightarrow$ c = 1 1

$\therefore$ Solution is $(2 - e^y)(x + 1) = 1$ $\frac{1}{2}$

22. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\,\sqrt{72}} = -1$ 1

$\Rightarrow \theta = 180°$ or π $\frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

23. Given lines are: $\dfrac{x-1}{-3} = \dfrac{y-2}{\left(\frac{\lambda}{7}\right)} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ 1

As lines are perpendicular,

$(-3)\left(\dfrac{-3\lambda}{7}\right) + \left(\dfrac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$ 1

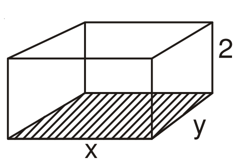
So, lines are

$\dfrac{x-1}{-3} = \dfrac{y-2}{1} = \dfrac{x-3}{2}$ and $\dfrac{x-1}{-3} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ $\frac{1}{2}$

Consider $\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

24. V = 2xy $\Rightarrow$ 2xy = 8 (given) 1



$\Rightarrow y = \dfrac{4}{x}$

Now, cost, $C = 70xy + 45 \times 2 \times (2x + 2y)$ 1

$$= 280 + 180x + \frac{720}{x}$$ 1

$$\frac{dC}{dx} = 180 - \frac{720}{x^2}$$ 1

$$\frac{dC}{dx} = 0 \Rightarrow x = 2m$$ $\frac{1}{2}$

$$\frac{d^2C}{dx^2} = \frac{1440}{x^3} = 180 > 0 \text{ at } x = 2$$ $\frac{1}{2}$

$\Rightarrow$ C is minimum at $x = 2m$. $\frac{1}{2}$

Minimum cost $= 280 + 180(2) + \frac{720}{2} = ₹\ 1{,}000$ $\frac{1}{2}$

25. $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ 2

$$\therefore\ A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$\therefore\ X = A^{-1} \cdot B$ 1

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$ 1

$\Rightarrow x = 3,\ y = 1,\ z = 2$ $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_2 \to R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2/5 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2,\ R_3 \to R_3 + 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 5R_3$

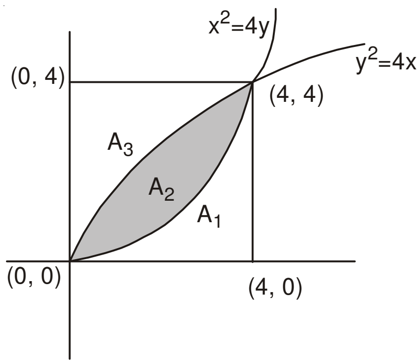
$$\Rightarrow \begin{bmatrix} 1 & 0 & -6/5 \\ 0 & 1 & -2/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 & 0 \\ 1/5 & 1/5 & 0 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{6}{5}R_3,\ R_2 \to R_2 + \frac{2}{5}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

26. Correct Figure 1



Point of intersection are (0, 0) and (4, 4) 1

here, $A_1 = \int_0^4 \frac{x^4}{4} dx = \frac{16}{3}$ ...(1) 1

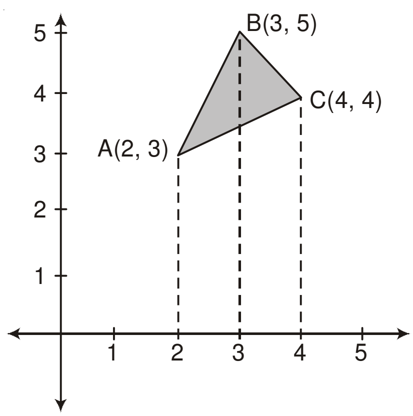
$A_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \frac{16}{3}$ ..(2) $1\frac{1}{2}$

$A_3 = \int_0^4 \frac{y^2}{4} dy = \frac{16}{3}$ ...(3) 1

From (1), (2) and (3), $A_1 = A_2 = A_3$. $\frac{1}{2}$

OR

Correct Figure 1



$$\left.\begin{array}{l} \text{Equation of } AB: y = 2x - 1 \\ \text{Equation of } BC: y = -x + 8 \\ \text{Equation of } AC: y = \frac{1}{2}(x+4) \end{array}\right\}$$ $1\frac{1}{2}$

Required Area $= \int_2^3 (2x-1)dx + \int_3^4 (-x+8)dx - \int_2^4 \left(\frac{x+4}{2}\right)dx$ $1\frac{1}{2}$

$= \left[x^2 - x\right]_2^3 + \left[\frac{-x^2}{2} + 8x\right]_3^4 - \frac{1}{2}\left[\frac{x^2}{2} + 4x\right]_2^4$ $1\frac{1}{2}$

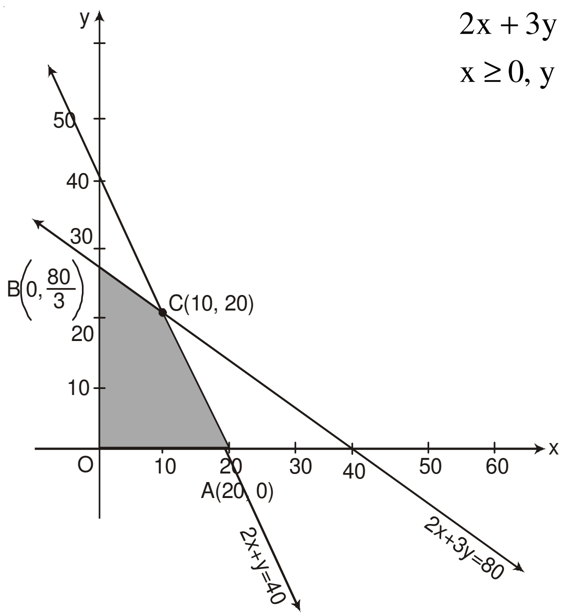
$= 4 + \frac{9}{2} - 7 = \frac{3}{2}$ $\frac{1}{2}$

27. Let number of items produced of model A be x and that of model B be y.

LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to



$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 & \end{array}\right\}$$ 2

Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
$B\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10,\ y = 20$. $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.

28. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i} + \hat{j} + \hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) = $\dfrac{|-2+1+4|}{\sqrt{(-1)^2+1^2+1^2}} = \sqrt{3}$ 1

29. X = no. of kings = 0, 1, 2 $\frac{1}{2}$

$P(X = 0) = P(\text{no king}) = \dfrac{48}{52} \times \dfrac{47}{51} = \dfrac{188}{221}$ 1

$P(X = 1) = P(\text{one king and one non-king}) = \dfrac{4}{52} \times \dfrac{48}{51} \times 2 = \dfrac{32}{221}$ 1

$P(X = 2) = P(\text{two kings}) = \dfrac{4}{52} \times \dfrac{3}{51} = \dfrac{1}{221}$ 1

Probability distribution is given by

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$\frac{1}{2}$

Now, Mean = $\Sigma X \cdot P(X) = \dfrac{34}{221}$ or $\dfrac{2}{13}$ 1

and $\text{Var}(X) = \Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2 = \frac{6800}{48841} \text{ or } \frac{400}{2873}$$ 1