QUESTION PAPER CODE 30/4/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Let nth term of the A.P. be 101.

$\therefore \quad t_n = -4 + (n - 1)3 = 101$ $\frac{1}{2}$

$3n - 7 = 101$

$n = \frac{108}{3} = 36$ $\frac{1}{2}$

2. $\frac{\tan 65°}{\cot 25°} = \frac{\tan(90° - 25°)}{\cot 25°}$ $\frac{1}{2}$

$= \frac{\cot 25°}{\cot 25°} = 1$ $\frac{1}{2}$

OR

$\sin 67° + \cos 75° = \sin (90° - 23°) + \cos (90° - 15°)$ $\frac{1}{2}$

$= \cos 23° + \sin 15°$ $\frac{1}{2}$

3. For equal roots, $4k^2 - 4k \times 6 = 0$ $\frac{1}{2}$

Hence $k = 6$ $\frac{1}{2}$

4. Here $1.41 < x < 2.6$

Any rational number lying between 1.4 ... & 2.6 ... 1

(variable answer)

OR

$2^2 \times 5^2 \times 5 \times 3^2 \times 17 = (10)^2 \times 5 \times 3^2 \times 17$

$\therefore$ No. of zeroes in the end of the number = Two 1

5. Required distance = $\sqrt{(-a-a)^2 + (-b-b)^2}$ $\frac{1}{2}$

$= \sqrt{4(a^2 + b^2)}$ or $2\sqrt{a^2 + b^2}$ $\frac{1}{2}$

6. Here $\frac{BC}{EF} = \frac{8}{11}$ $\frac{1}{2}$

$\therefore$ BC = $\frac{8}{11} \times 15.4 = 11.2$ cm $\frac{1}{2}$

SECTION B

7. $\frac{3}{x} + \frac{8}{y} = -1$ ...(i)

$\frac{1}{x} - \frac{2}{y} = 2$ ...(ii)

Multiply (ii) by 3 and subtract from (i), we get

$\frac{14}{y} = -7 \Rightarrow y = -2$ 1

Substitute this value of y = –2 in (i), we get x = 1

Hence, x = 1, y = –2 1

OR

For unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \neq \frac{2}{6}$ 1

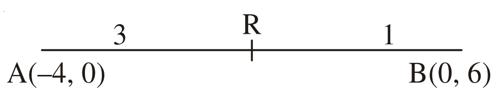
$\Rightarrow k \neq 1$ 1

The pair of equations have unique solution for all real values of k except 1.

8. $\left.\begin{aligned} 867 &= 3 \times 255 + 102 \\ 255 &= 2 \times 102 + 51 \\ 102 &= 2 \times 51 + 0 \end{aligned}\right\}$ $1\frac{1}{2}$

$\therefore$ HCF = 51 $\frac{1}{2}$

9.



$\frac{AR}{AB} = \frac{3}{4} \Rightarrow \frac{AR}{RB} = \frac{3}{1}$ 1

$\therefore$ R = $\left(\frac{3\times 0 + 1(-4)}{4}, \frac{3\times 6 + 1\times 0}{4}\right)$, i.e., $\left(-1, \frac{9}{2}\right)$ 1

10. 12, 16, 20, ..., 204 $\frac{1}{2}$

Let the number of multiples be n.

$\therefore\ t_n = 12 + (n - 1) \times 4 = 204$ 1

$\Rightarrow n = 49$ $\frac{1}{2}$

OR

Here $t_3 = 16$ and $t_7 = t_5 + 12$ $\frac{1}{2}$

$\Rightarrow\ a + 2d = 16$ (i) and $a + 6d = a + 4d + 12$ (ii) $\frac{1}{2}$

From (ii), d = 6

From (i), a = 4 1

$\therefore$ A.P. is 4, 10, 16, ...

11. The possible number of outcomes are 8 {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} 1

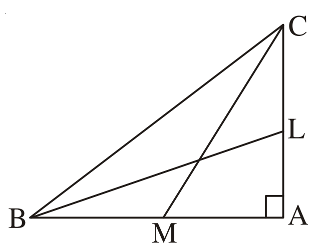
P (exactly one head) = $\frac{3}{8}$ 1

12. (a) P(a prime no.) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

(b) P(odd no.) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

SECTION C

13. In right angled triangle CAM,



$$CM^2 = CA^2 + AM^2 \quad \text{...(i)}$$

Similarly, $$BC^2 = AC^2 + AB^2 \quad \text{...(ii)}$$

and $$BL^2 = AL^2 + AB^2 \quad \text{...(iii)}$$ 1

Now $4(BL^2 + CM^2) = 4(AL^2 + AB^2 + AC^2 + AM^2)$ 1

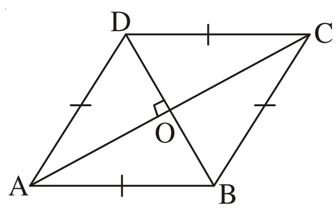
But $AL = LC = \frac{1}{2}AC$ and $AM = MB = \frac{1}{2}AB$

$$\therefore\ 4(BL^2 + CM^2) = 4\left(\frac{AC^2}{4} + AB^2 + AC^2 + \frac{AB^2}{4}\right)$$

$$= 4\left(\frac{5}{4}AB^2 + \frac{5}{4}AC^2\right)$$

$= 5(AB^2 + AC^2) = 5BC^2$ 1

OR



Let ABCD be rhombus and its diagonals intersect at O.

In ΔAOB, $AB^2 = AO^2 + OB^2$ 1

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$= \frac{1}{4}(AC^2 + BD^2)$ 1

$\Rightarrow 4AB^2 = AC^2 + BD^2$

$\Rightarrow AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$ (ABCD being rhombus) 1

14. Area of shaded region

$= \left[\pi(42)^2 - \pi(21)^2\right]\frac{300°}{360°}$ 1

$= \frac{22}{7} \times 63 \times 21 \times \frac{5}{6}.$ 1

$= 3465 \text{ cm}^2$ 1

15. Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2 \times 24 \text{ cm}^3$ 1

Let the radius of the sphere be R cm

$\therefore \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times 36 \times 24$ 1

$\Rightarrow R^3 = 6 \times 6 \times 6$

$\Rightarrow R = 6$ cm $\frac{1}{2}$

Surface area = $4\pi R^2 = 144\pi \text{ cm}^2$ $\frac{1}{2}$

OR

Water required to fill the tank = $\pi(5)^2 \times 2 = 50\pi \text{ m}^3$ 1

Water flown in 1 hour = $\pi\left(\frac{1}{10}\right)^2 \times 3000 \text{ m}^3$

$= 30\pi \text{ m}^3$ 1

Time taken to fill $30\pi \text{ m}^3$ = 60 minutes

Time taken to fill $50\pi \text{ m}^3 = \frac{60}{30} \times 50 = 100$ minutes 1

16. Here the modal class is 20 – 25 $\frac{1}{2}$

Mode $= 20 + \frac{20-7}{40-7-8} \times 5$ 2

$= 20 + \frac{13}{25} \times 5 = 22.6$ Hence mode = 22.6 $\frac{1}{2}$

17. Let $\frac{2+3\sqrt{2}}{7}$ be a rational number say 'a'

$\therefore \quad \frac{2+3\sqrt{2}}{7} = a$ 1

$\Rightarrow \quad 3\sqrt{2} = 7a - 2$

$\Rightarrow \quad \sqrt{2} = \frac{7a-2}{3}$ 1

This is a contradiction because $\sqrt{2}$ is an irriational number and $\frac{7a-2}{3}$ is a rational number. 1

Hence $\frac{2+3\sqrt{2}}{7}$ is an irrational number.

18. The polynomial whose zeroes are 2 and –2 is

$(x - 2)(x + 2)$ i.e. $x^2 - 4$ 1

$\therefore \quad 2x^4 - 5x^3 - 11x^2 + 20x + 12 = (x^2 - 4)(2x^2 - 5x - 3)$ 1

$= (x + 2)(x - 2)(2x + 1)(x - 3)$

$\therefore$ Zeroes are 2, –2, 3 and $-\frac{1}{2}$ 1

19. Let the speed of stream = x km/hr.

$$\therefore \quad \frac{24}{18-x} - \frac{24}{18+x} = 1$$ $1\frac{1}{2}$

$\Rightarrow \; x^2 + 48x - 324 = 0$ 1

$\Rightarrow \; (x - 6)(x + 54) = 0$

$\Rightarrow \; x = 6$

i.e. speed of stream = 6 km/hr $\frac{1}{2}$

20. LHS $= (\sin\theta + \cos\theta + 1)(\sin\theta + \cos\theta - 1)\sec\theta \operatorname{cosec}\theta$

$= [(\sin\theta + \cos\theta)^2 - 1]\sec\theta \operatorname{cosec}\theta$ 1

$= 2\sin\theta\cos\theta\sec\theta\operatorname{cosec}\theta$ 1

$= 2 = \text{RHS}$ 1

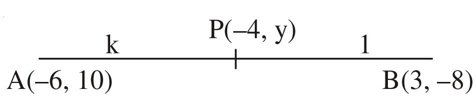
OR

$$\text{LHS} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{\sec^2\theta - 1}}$$ 1

$$= \frac{2\sec\theta}{\tan\theta}$$ 1

$$= \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta = \text{RHS}$$ 1

21.



Let point P divides the line segment AB in the ratio k : 1

$$\therefore \quad \frac{3k-6}{k+1} = -4$$ 1

$\Rightarrow \; 3k - 6 = -4k - 4$

$\Rightarrow \; 7k = 2$ i.e., $k = \frac{2}{7}$ $\therefore$ Ratio is 2 : 7 1

Again $\frac{2\times(-8) + 7\times 10}{2+7} = y \Rightarrow y = 6$ 1

Hence y = 6

OR

The points are collinear if the area of triangle formed is zero.

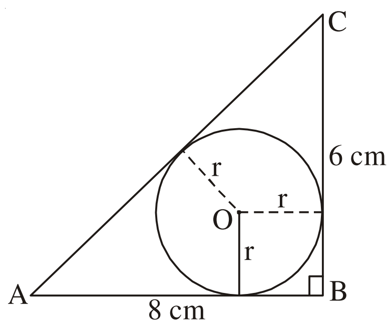
i.e., $-5(p + 2) + 1(-2 - 1) + 4(1 - p) = 0$ $1\frac{1}{2}$

$-5p - 10 - 3 + 4 - 4p = 0$

$-9p = 9$

$p = -1$ $1\frac{1}{2}$

22.



$AC = \sqrt{AB^2 + BC^2} = \sqrt{64 + 36} = 10 \text{ cm}$ $\frac{1}{2}$

Area of $\Delta ABC = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$ $\frac{1}{2}$

Let r be the radius of inscribed circle.

$ar(\Delta ABC) = ar(AOB) + ar(\Delta BOC) + ar(\Delta AOC)$

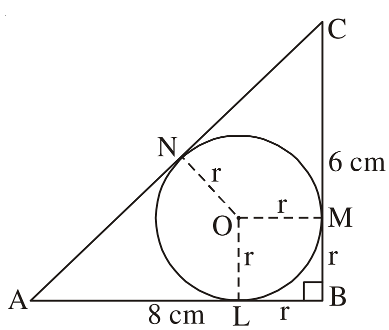
$= \frac{1}{2} \times 8r + \frac{1}{2} \times 6r + \frac{1}{2} \times 10r$ 1

$= \frac{1}{2} r(8 + 6 + 10) = 12r$

$12r = 24 \Rightarrow r = 2$ cm $\frac{1}{2}$

$\therefore$ Diameter = 4 cm $\frac{1}{2}$

Alternate method:



Here BL = BM = r (sides of squares)

$AC = \sqrt{AB^2 + BC^2} = 10$ cm 1

$AL = AN = 8 - r$ and $CM = CN = 6 - r$ $\frac{1}{2}$

$AC = AN + NC$

$\Rightarrow 10 = 8 - r + 6 - r$

$\Rightarrow 2r = 4$

$\Rightarrow r = 2$ $\frac{1}{2}$

$\therefore$ Diameter = 4 cm 1

SECTION D

23. Here $a_1 = -4$, $a_n = 29$ and $S_n = 150$

Now $29 = -4 + (n-1)d = (n-1)d = 33$ ...(i) $1\frac{1}{2}$

Also $S_n = 150 = \frac{n}{2}(-4+29) \Rightarrow n = 12$ $1\frac{1}{2}$

From (i), d = 3

Hence common difference = 3 1

24. Drawing circle of radius 4 cm and taking a point 6 cm away from the centre $1\frac{1}{2}$

Drawing two tangents 2

Length of tangents = 4.5 cm (approx.) $\frac{1}{2}$

25. LHS $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\cos^2\theta\sin^2\theta - 2\cos^2\theta\sin^2\theta] + 1$ 1

$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)] - 3[(\sin^2\theta + \cos^2\theta)^2 - 2\cos^2\theta\sin^2\theta] + 1$

$= 2(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) - 3(1 - 2\cos^2\theta\sin^2\theta) + 1$ 1

$= 2[(\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta] - 3(1 - 2\cos^2\theta\sin^2\theta) + 1$

$= 2(1 - 3\sin^2\theta\cos^2\theta) - 3(1 - 2\cos^2\theta\sin^2\theta) + 1$ 1

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1$

$= 0 =$ RHS 1

26. $\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

or $\frac{2x-2a-b-2x}{2x(2a+b+2x)} = \frac{b+2a}{2ab}$ 1

or $\frac{-(2a+b)}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$ 1

or $2x^2 + x(2a+b) + ab = 0$

$(x+a)(2x+b) = 0$ 1

$\Rightarrow x = -a$ or $-\frac{b}{2}$ 1

OR

Let x and y be lengths of the sides of two squares.

$\therefore\ x^2 + y^2 = 640$ and $4(x - y) = 64$ i.e., $x - y = 16$ 1

$x^2 + (x - 16)^2 = 640$ 1

or $x^2 + x^2 - 32x + 256 - 640 = 0$

or $2x^2 - 32x - 384 = 0$

or $x^2 - 16x - 192 = 0$

or $(x + 8)(x - 24) = 0 \Rightarrow x = 24$ 1

$\therefore\ y = x - 16 = 24 - 16 = 8$

Hence lengths of sides of the squares are 24 cm and 8 cm. 1

27. In ΔABD, $AB^2 = AD^2 + BD^2 \Rightarrow AD^2 = AB^2 - BD^2$ 1

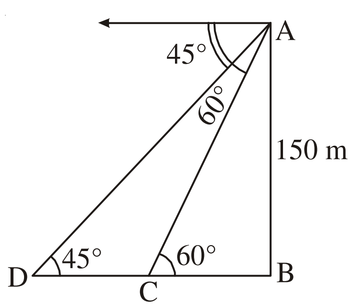
In ΔADC, $AC^2 = AD^2 + CD^2$

$= AB^2 - BD^2 + (BC - BD)^2$ 1

$= AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$ 1

$= AB^2 + BC^2 - 2BC \times BD$ 1

28. Correct Figure 1



$\frac{150}{BC} = \tan 60° = \sqrt{3}$

$\Rightarrow\ BC = \frac{150}{\sqrt{3}} = 50\sqrt{3}$ m $\frac{1}{2}$

Also $\frac{AB}{BD} = \tan 45° = 1 \Rightarrow AB = BD = 150$ m $\frac{1}{2}$

Now $CD = BD - BC = (150 - 50\sqrt{3})$ m $\frac{1}{2}$

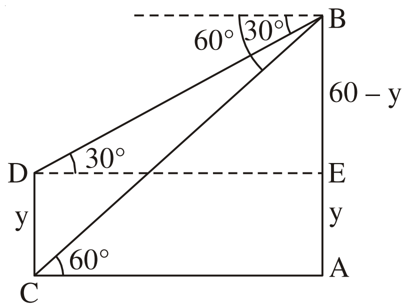
Distance travelled in 2 minutes = $(150 - 50\sqrt{3})$ m

$\therefore$ Distance travelled in 1 minute = $(75 - 25\sqrt{3})$ m 1

or $75 - 25(1.732) = 75 - 43.3 = 31.7$ m/minute

Hence speed of boat is $(75 - 25\sqrt{3})$ m/minutes or 31.7 m/minutes $\frac{1}{2}$

OR



Correct Figure 1

In ΔABC, $\frac{AB}{AC} = \tan 60°$

$\frac{60}{AC} = \sqrt{3}$

$AC = 20\sqrt{3}$ m 1

In ΔBED, $\frac{60-y}{DE} = \tan 30° = \frac{1}{\sqrt{3}}$ 1

i.e., $\frac{60-y}{20\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow 60 - y = 20$ i.e., $y = 40$ m $\frac{1}{2}$

Hence width of river $= 20\sqrt{3}$ m and height of other pole = 40 m $\frac{1}{2}$

29.

Classes	Class mark (X)	Frequency (f_i)	$f_i x_i$	
10-30	20	5	100	
30-50	40	8	320	
50-70	60	12	720	
70-90	80	20	1600	Correct Table 2
90-110	100	3	300	
110-130	120	2	240	

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3280}{50} = 65.6$$ 2

Alternate methods by assuming mean are acceptable.

OR

	cf	
More than or equal to 65	24	
More than or equal to 60	54	
More than or equal to 55	74	Table $1\frac{1}{2}$
More than or equal to 50	90	
More than or equal to 45	96	
More than or equal to 40	100	

Plotting graph of (40, 100), (45, 96), (50, 90), (55, 74), (60, 54)

and (65, 24) and joining the points $1\frac{1}{2}+1$

30. Volume of the container $= \frac{\pi}{3}h(r_1^2 + r_2^2 + r_1r_2)$

$= \frac{3.14}{3} \times 16\,(20^2 + 8^2 + 20 \times 8)$ $\frac{1}{2}$

$= 3.14 \times 16 \times 208 = 10450 \text{ cm}^3$ 1

$= 10.45$ litres

Cost of milk = 10.45 × 50 = ₹ 522.50 $\frac{1}{2}$

Slant height of frustum $= \sqrt{16^2 + 12^2} = 20\,\text{cm}$ $\frac{1}{2}$

Surface area $= \pi[(r_1 + r_2)l + r_2^2]$

$= 3.14[(8 + 20)\ 20 + 8^2]$

$= 3.14 \times 624 = 1959.36 \text{ cm}^2$ 1

$\therefore$ Cost of metal used $= \frac{10}{100} \times 1959.36 =$ ₹195.93 $\frac{1}{2}$