Marking Scheme

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Senior School Certificate Examination, 2023

MATHEMATICS PAPER CODE 65/3/2

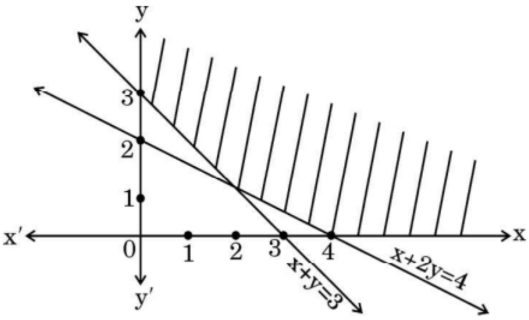
General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives the impression that answer is correct, and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

14	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

	SECTION A	
Q. No.	Expected Answers/Value Points	Marks
Q1	$\int 2^{x+2}dx$ is equal to : (a) $2^{x+2} + C$ (b) $2^{x+2} \log 2 + C$ (c) $\frac{2^{x+2}}{\log 2} + C$ (d) $2 \cdot \frac{2^{x}}{\log 2} + C$	
Ans	(c) $\frac{2^{x+2}}{\log 2} + C$	1
Q2	Let A be a skew-symmetric matrix of order 3. If $\lvert A \rvert = x$, then $(2023)^x$ is equal to : (a) 2023 (b) $\frac{1}{2023}$ (c) $(2023)^2$ (d) 1	
Ans	**(d) 1**	1
Q3	$\int_0^2 \sqrt{4 - x^2}\, dx$ equals : (a) 2 log 2 (b) – 2 log 2 (c) $\frac{\pi}{2}$ (d) π	
Ans	(d) π	1
Q4	The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is : (a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $\log x - \log y = C$ (c) $xy = C$ (d) $x + y = C$	
Ans	(c) $xy = C$	1
Q5	What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2} \sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$? (a) 3 (b) 2 (c) 6 (d) not defined	
Ans	**(b) 2**	1

Q6	The direction cosines of vector $\overrightarrow{BA}$, where coordinates of A and B are (1, 2, – 1) and (3, 4, 0) respectively, are : (a) $-2, -2, -1$ (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (c) $2, 2, 1$ (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$	
Ans	(b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$	1
Q7	$\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is : (a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0	
Ans	(a) $\frac{\pi}{2}$	1
Q8	In Δ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to : (a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$ (c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$	
Ans	(d) $2\hat{i} + 3\hat{k}$	1
Q9	If the point P(a, b, 0) lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is : (a) (1, 2) (b) $\left(\frac{1}{2}, \frac{2}{3}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (d) (0, 0)	
Ans	(c) $\left(\frac{1}{2}, \frac{1}{4}\right)$	1
Q10	For any two events A and B, if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ equals : (a) $\frac{3}{8}$ (b) $\frac{8}{9}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$	

Ans	**One mark should be awarded to everyone who has attempted this question.**	1
Q11	The value of k for which function $f(x)=\begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is : (a) 1 (b) 2 (c) any real number (d) 0	
Ans	**(d) 0**	1
Q12	If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is : (a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$ (c) $\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\|$ (d) $-\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\|$	
Ans	(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$	1
Q13	The number of feasible solutions of the linear programming problem given as Maximize z = 15x + 30y subject to constraints : $3x + y \leq 12,\ x + 2y \leq 10,\ x \geq 0, y \geq 0$ is (a) 1 (b) 2 (c) 3 (d) infinite	
Ans	**(d) infinite**	1
Q14	The feasible region of a linear programming problem is shown in the figure below :  Which of the following are the possible constraints ? (a) $x + 2y \geq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$ (b) $x + 2y \leq 4,\ x + y \leq 3,\ x \geq 0,\ y \geq 0$ (c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$ (d) $x + 2y \geq 4,\ x + y \geq 3,\ x \leq 0,\ y \leq 0$	
Ans	(c) $x + 2y \geq 4,\ x + y \geq 3,\ x \geq 0,\ y \geq 0$	1

Q15	If $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}$ and $B=\begin{bmatrix}1&1\\0&0\end{bmatrix}$, then $B'A'$ is equal to : (a) $\begin{bmatrix}1&1\\0&0\end{bmatrix}$ (b) $\begin{bmatrix}1&0\\1&0\end{bmatrix}$ (c) $\begin{bmatrix}1&1\\1&1\end{bmatrix}$ (d) $\begin{bmatrix}0&0\\0&0\end{bmatrix}$	
Ans	(b) $\begin{bmatrix}1&0\\1&0\end{bmatrix}$	1
Q16	If $A . (\text{adj } A) = \begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$, then the value of $\lvert A\rvert + \lvert \text{adj } A\rvert$ is equal to : (a) 12 (b) 9 (c) 3 (d) 27	
Ans	**(a) 12**	1
Q17	A and B are skew-symmetric matrices of same order. AB is symmetric, if : (a) AB = O (b) AB = – BA (c) AB = BA (d) BA = O	
Ans	(c) AB = BA	1
Q18	For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\, I$, where $A = \begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$? (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) 0 (d) $\frac{\pi}{2}$	
Ans	(b) $\frac{\pi}{6}$	1

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

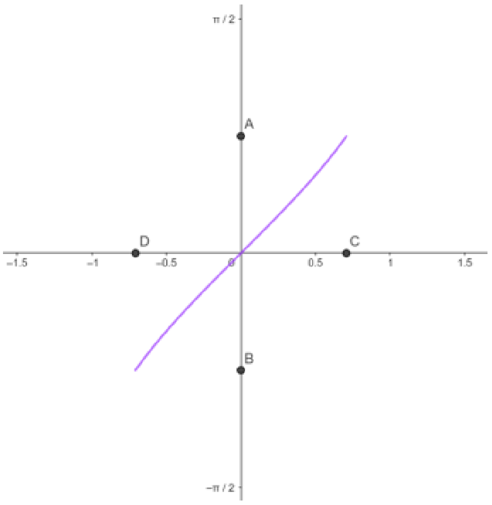
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

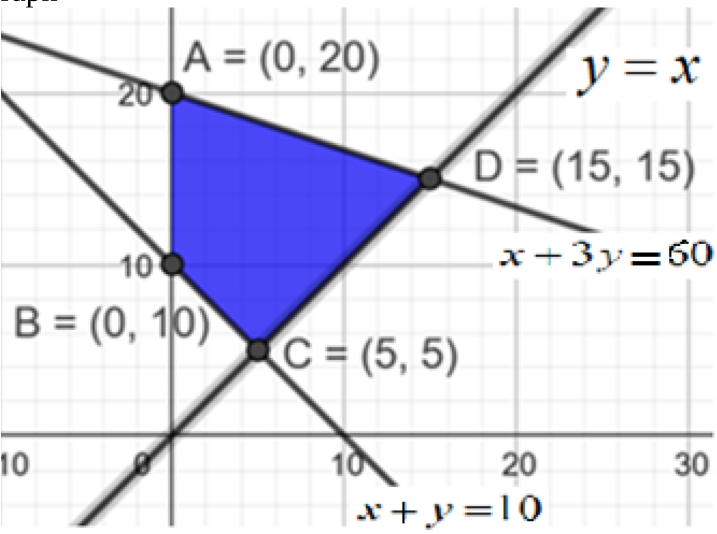
(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Q19	*Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5). *Reason (R):* Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.	
Ans	(c) Assertion (A) is true and Reason (R) is false.	1
Q20	*Assertion (A) :* Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$. *Reason (R) :* Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.	
Ans	(d) Assertion (A) is false and Reason (R) is true.	1
	SECTION B	
Q21	Consider the statement "There exists at least one value of $b \in \mathbb{R}$ for which $f(x) = \frac{b}{x}$, $b \neq 0$ is strictly increasing in $\mathbb{R} - \{0\}$." State True or False. Justify.	
Ans	The given statement is "True". $f'(x) = -\frac{b}{x^2}$ for $b < 0$, $f'(x) > 0$ in $(-\infty, 0)$ and $(0, \infty)$ $\therefore$ f(x) is strictly increasing in both these intervals.	1 ½ ½
Q22(a)	Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$	
Ans	(a) Given expression $= \frac{3\pi}{4} + \frac{2\pi}{6} + \frac{\pi}{2}$ $= \frac{19\pi}{12}$ **OR**	1½ ½
Q22(b)	Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).	
Ans	(b)	

	 Correct graph Here, the points A, B, C and D are respectively $\left(0, \frac{\pi}{4}\right), \left(0, -\frac{\pi}{4}\right), \left(\frac{1}{\sqrt{2}}, 0\right), \left(-\frac{1}{\sqrt{2}}, 0\right)$. Range = $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$	1 1
Q23(a)	If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.	
Ans	(a) $y = x^{1/x}$ $\Rightarrow \log y = \frac{1}{x} \log x$ $\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{\log x}{x^2} + \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}} \frac{(1 - \log x)}{x^2}$ $\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 1$ **OR**	½ 1 ½
Q23(b)	If $x = a \sin 2t, y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.	
Ans	(b) $\frac{dx}{dt} = 2a \cos 2t$ $\frac{dy}{dt} = 2a\left(-\sin 2t + \frac{\sec^2 t}{2 \tan t}\right) = 2a \frac{\cos^2 2t}{\sin 2t}$ $\frac{dy}{dx} = \cot 2t$	½ 1 ½
Q24	If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.	
Ans	$(\vec{r} \times \hat{j}) . (\vec{r} \times \hat{k}) - 12 = (3\hat{k} - 6\hat{i}) . (-3\hat{j} - 2\hat{i}) - 12$ $= 12 - 12 = 0$	$1\frac{1}{2}$ ½

Q25	Find the value of p, so that lines $\frac{x-1}{-2}=\frac{y-4}{3p}=\frac{z-3}{4}$ and $\frac{x-2}{4p}=\frac{y-5}{2}=\frac{1-z}{7}$ are perpendicular to each other.	
Ans	d.r.'s of lines are < – 2, 3p, 4 > and < 4p, 2, – 7 > As lines are perpendicular $-8p+6p-28=0$ $\Rightarrow p=-14$	1 ½ ½
	SECTION C	
Q26	Find : $\int \frac{e^x}{\sqrt{e^{2x}-4e^x-5}}dx$	
Ans	Let $e^x = t$. Then $e^x\,dx = dt$ Given integral becomes $\int \frac{dt}{\sqrt{t^2-4t-5}}$ $=\int \frac{dt}{\sqrt{(t-2)^2-3^2}}$ $=\log \lvert (t-2)+\sqrt{t^2-4t-5} \rvert + C$ $=\log \lvert e^x-2+\sqrt{e^{2x}-4e^x-5} \rvert + C$	½ 1 1 ½
Q27(a)	Find: $\int \frac{\cos x}{\sin 3x}dx$	
Ans	(a) $I=\int \frac{\cos x}{3\sin x-4\sin^3 x}dx$ Let $\sin x = t \Rightarrow \cos x\,dx = dt$ $I=\int \frac{dt}{3t-4t^3}$ $=\int \frac{1}{t^3\left(\frac{3}{t^2}-4\right)}dt$ Let $\frac{3}{t^2}-4=z \Rightarrow -\frac{6}{t^3}dt = dz$ $I=-\frac{1}{6}\int \frac{dz}{z}$	 ½ ½ ½ ½

	$= -\frac{1}{6}\log\lvert z\rvert + C$ $= -\frac{1}{6}\log\lvert 3\operatorname{cosec}^2 x - 4\rvert + C$ **OR**	½ ½
Q27(b)	Find: $\int x^2 \log(x^2+1)\,dx$	
Ans	(b)Let I = $\int x^2 \log(x^2+1)dx$ $= \log(x^2+1)\,.\,\frac{x^3}{3} - \int \frac{2x}{x^2+1}\,.\,\frac{x^3}{3}dx$ $= \frac{x^3}{3}\log(x^2+1) - \frac{2}{3}\int \frac{x^4}{x^2+1}dx$ $= \frac{x^3}{3}\log(x^2+1) - \frac{2}{3}\int\left(x^2 - 1 + \frac{1}{x^2+1}\right)dx$ $= \frac{x^3}{3}\log(x^2+1) - \frac{2}{3}\left[\frac{x^3}{3} - x + \tan^{-1}x\right] + C$	1 ½ ½ 1
Q28	Solve the following linear programming problem graphically : Maximize $z = 3x + 9y$ subject to the constraints $x + y \geq 10$ $x + 3y \leq 60$ $x \leq y$ $x \geq 0, y \geq 0$	
Ans	Correct graph 	2

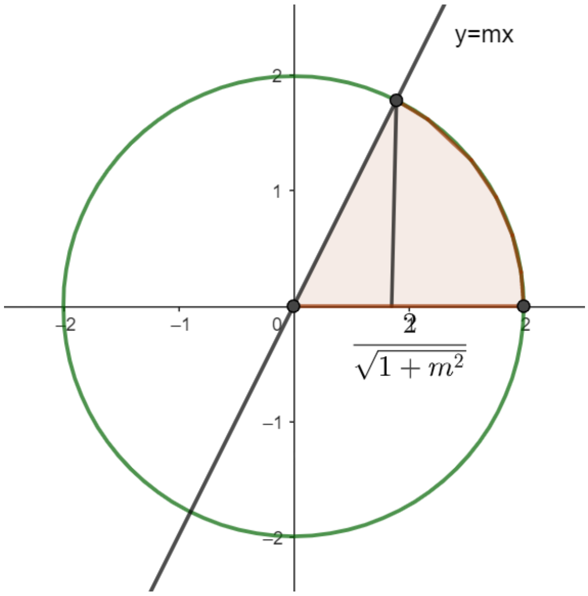
<table>
<tr><td></td><td>
<table>
<tr><td>Corner points</td><td>Value of Z = 3x + 9y</td></tr>
<tr><td>A(0, 20)</td><td>180 → Maximum</td></tr>
<tr><td>B(0, 10)</td><td>90</td></tr>
<tr><td>C(5, 5)</td><td>60</td></tr>
<tr><td>D(15, 15)</td><td>180 → Maximum</td></tr>
</table>
Maximum lies at every point on the line segment AD.</td><td>1</td></tr>
<tr><td>Q29(a)</td><td colspan="2">(a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.</td></tr>
<tr><td>Ans</td><td>(a)
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>P(X)</td><td>$\frac{6}{36}$</td><td>$\frac{10}{36}$</td><td>$\frac{8}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{2}{36}$</td></tr>
</table>
OR</td><td>1 ½
1 ½</td></tr>
<tr><td>Q29(b)</td><td colspan="2">There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.</td></tr>
<tr><td>Ans</td><td>(b)E_1 = Biased coin is selected $\Rightarrow P(E_1)=\frac{1}{2}$
E_2 = Fair coin is selected $\Rightarrow P(E_2)=\frac{1}{2}$
A = Head appeared on tossing a selected coin .
$P\left(\frac{A}{E_1}\right)=\frac{1}{4}$, $P\left(\frac{A}{E_2}\right)=\frac{1}{2}$
By Bayes' Theorem $P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$
$= \frac{\frac{1}{2}\cdot\frac{1}{4}}{\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{2}}$
$= \frac{1}{3}$</td><td>½
1
1
½</td></tr>
</table>

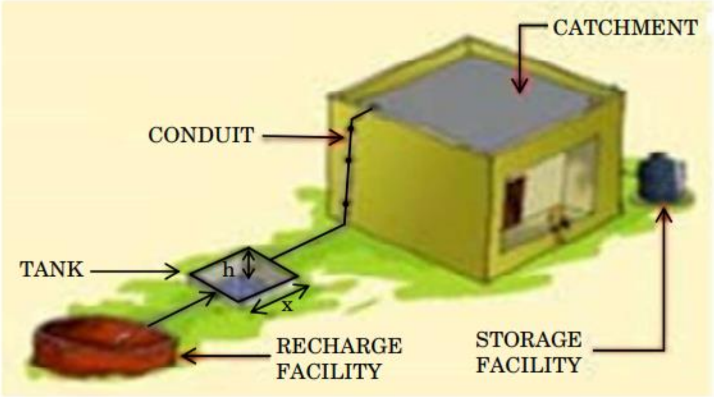
Q30(a)	Find the general solution of the differential equation : $\frac{d}{dx}\left(xy^2\right) = 2y\left(1 + x^2\right)$	
Ans	(a) Given differential equation is $2xy\frac{dy}{dx} + y^2 = 2y(1 + x^2)$	½
	$\Rightarrow \frac{dy}{dx} + \frac{y}{2x} = \frac{1}{x} + x$	
	Integrating factor = $e^{\int \frac{1}{2x}dx} = e^{\log\sqrt{x}} = \sqrt{x}$	1
	Solution is given by $y\sqrt{x} = \int\left(\frac{1}{\sqrt{x}} + x^{\frac{3}{2}}\right)dx$	1
	$\Rightarrow y\sqrt{x} = 2\sqrt{x} + \frac{2x^{\frac{5}{2}}}{5} + C$,or $y = 2 + \frac{2x^2}{5} + \frac{C}{\sqrt{x}}$	½
	OR	
Q30(b)	Solve the following differential equation : $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$	
Ans	(b) Given differential equation is $\frac{dy}{dx} = \frac{y}{x} - e^{\frac{y}{x}}$	½
	Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$	½
	The given equation becomes $v + x\frac{dv}{dx} = v - e^{v}$	
	$\Rightarrow - e^{-v}dv = \frac{dx}{x}$	½
	Integrating both sides, we get	
	$e^{-v} = \log\lvert x\rvert + C$	1
	$\Rightarrow e^{-\frac{y}{x}} = \log\lvert x\rvert + C$	½
Q31	Evaluate : $\int_{-\pi/2}^{\pi/2} \frac{\sin^{100}x}{\sin^{100}x + \cos^{100}x}\,dx$	
Ans	$I = \int_{-\pi/2}^{\pi/2} \frac{sin^{100}\ x}{\sin^{100}x + \cos^{100}x}dx$	
	$I = 2\int_0^{\pi/2} \frac{sin^{100}\ x}{\sin^{100}x + \cos^{100}x}dx$ as $f(x) = \frac{sin^{100}\ x}{\sin^{100}x + \cos^{100}x}$ is even	½
	$I = 2\int_0^{\pi/2} \frac{cos^{100}\ x}{\cos^{100}x + \sin^{100}x}dx$ using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$	1
	$2I = 2\int_0^{\pi/2} \frac{sin^{100}\ x + \cos^{100}x}{cos^{100}\ x + \sin^{100}x}dx = 2\int_0^{\pi/2} dx$	1
	$I = x\rvert_0^{\pi/2} \quad \Rightarrow I = \frac{\pi}{2}$	½

	SECTION D	
Q32(a)	If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = 0$	
Ans	(a) getting, $A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$ getting, $A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$ $\therefore\ A^3 - 6A^2 + 7A + 2I =$ $\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$	$1\frac{1}{2}$ $1\frac{1}{2}$ 1 1
	OR	
Q32(b)	If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations : $3x + 5y = 11,\ 2x - 7y = -3.$	
Ans	(b)adj A = $\begin{bmatrix} -7 & -2 \\ -5 & 3 \end{bmatrix}$ $\lvert A \rvert = -31$ $A^{-1} = \frac{-1}{31}\begin{bmatrix} -7 & -2 \\ -5 & 3 \end{bmatrix}$ Given system of equations is $\begin{bmatrix} 3 & 5 \\ 2 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$ which is A′X = B, where $X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$ $\Rightarrow X = (A')^{-1}B$ $\Rightarrow X = (A^{-1})'B$ $= \frac{-1}{31}\begin{bmatrix} -7 & -5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 11 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\therefore\ x = 2,\quad y = 1$	1 1 ½ ½ ½ ½ 1

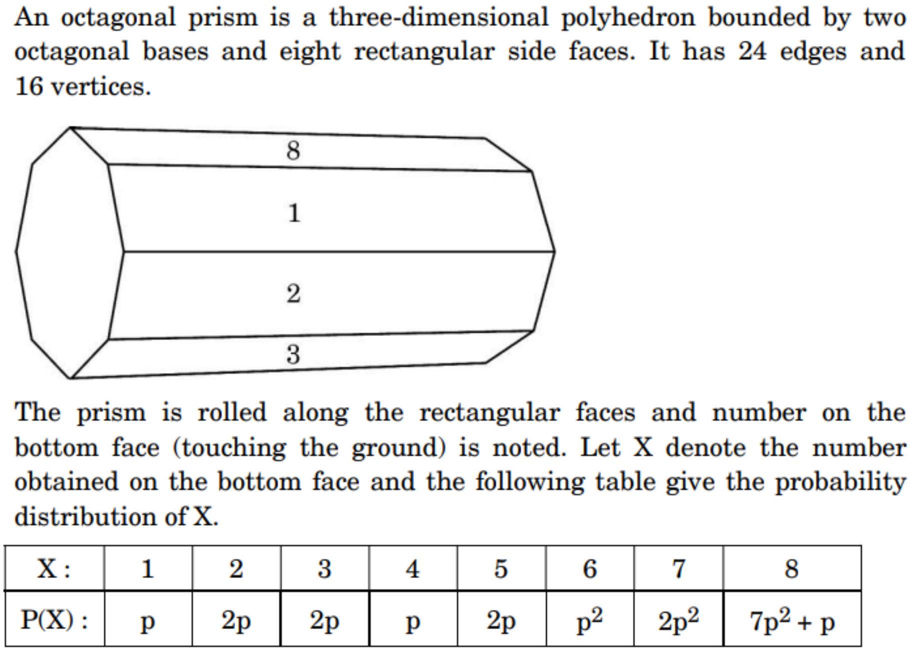
Q33(a)	Find the value of b so that the lines $\frac{x-1}{2}=\frac{y-b}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are intersecting lines. Also, find the point of intersection of these given lines.	
Ans	(a)As lines are intersecting, $(\vec{a_2}-\vec{a_1}).((\vec{b_1}\times\vec{b_2}) = 0$	
	$\Rightarrow \begin{vmatrix} 3 & 1-b & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix} = 0$	1
	$\Rightarrow$ b = 2	1
	Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$, $\lambda \in R$	1
	For the point of intersection, these coordinates must satisfy $\frac{x-4}{5} = \frac{y-1}{2} = z$ $\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = 4\lambda + 3$	1
	$\Rightarrow \lambda = -1$	½
	∴ point of intersection is (– 1, – 1, – 1)	½
	OR	
Q33(b)	Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.	
Ans	(b)Equation of the line AB : $\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4}$	½
	Equation of the line BC : $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{3}$	½
	Equation of the line CD : $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$	½
	Equation of the line DA : $\frac{x-4}{3} = \frac{y-7}{5} = \frac{z-8}{3}$	½
	Let P be foot of perpendicular from A to CD.	
	∴ Coordinates of P are $(\lambda - 1, 2\lambda - 2, 2\lambda + 1)$ for some λ	1

	d.r.'s of AP are $(\lambda - 5, 2\lambda - 9, 2\lambda - 7)$ since AP⊥CD $\Rightarrow 1(\lambda - 5) + 2(2\lambda - 9) + 2(2\lambda - 7) = 0$ $\Rightarrow 9\lambda = 37 \quad \Rightarrow \lambda = \frac{37}{9}$ $\therefore$ Coordinates of P are $\left(\frac{28}{9}, \frac{56}{9}, \frac{83}{9}\right)$	½ ½ ½ ½
Q34	Prove that a function f : $[0, \infty) \rightarrow [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.	
Ans	Let $x_1, x_2 \in [0, \infty)$ such that $f(x_1) = f(x_2)$ Then this $\Rightarrow 4x_1^2 + 4x_1 - 5 = 4x_2^2 + 4x_2 - 5$ $\Rightarrow (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)[(x_1 + x_2) + 1] = 0$ $\Rightarrow (x_1 - x_2) = 0$ or $x_1 + x_2 = -1$, which is rejected as $x_1, x_2 \geq 0$ $\Rightarrow x_1 = x_2$ $\therefore$ f is one-one. Let $f(x) = y \Rightarrow y = 4x^2 + 4x - 5$ for $x \in [0, \infty)$ $\Rightarrow 4x^2 + 4x - 5 - y = 0$ $\Rightarrow x = \frac{-4 \pm \sqrt{16 - 16(-5 - y)}}{8} \Rightarrow x = \frac{-4 + 4\sqrt{6 + y}}{8} = \frac{-1 + \sqrt{6 + y}}{2}$ Since, $x \geq 0$, we have $y + 6 \geq 1 \Rightarrow y \in [-5, \infty)$ $\therefore$ Range = Codomain $= [-5, \infty)$ Hence f is onto.	$2\frac{1}{2}$ $2\frac{1}{2}$
Q35	The area of the region bounded by the line $y = mx$ $(m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.	
Ans	Correct figure :	1

		
	$x^2 + y^2 = 4$ and $y = mx$	
	$\Rightarrow x^2 + m^2x^2 = 4 \ \Rightarrow x = \frac{2}{\sqrt{1 + m^2}}$	
	x- coordinate of the required point of intersection is $\frac{2}{\sqrt{1 + m^2}}$.	1
	According to question,	
	$\int_0^{\frac{2}{\sqrt{1+m^2}}} mx\,dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} \sqrt{4 - x^2}\,dx = \frac{\pi}{2}$	1+1
	$\Rightarrow m \left.\frac{x^2}{2}\right\vert_0^{\frac{2}{\sqrt{1+m^2}}} + \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\left.\frac{x}{2}\right\vert_{\frac{2}{\sqrt{1+m^2}}}^{2} = \frac{\pi}{2}$	½
	$\Rightarrow \frac{2m}{1+m^2} + \pi - \frac{2m}{1+m^2} - 2\sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{2}$	
	$\Rightarrow \frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{1+m^2}}$	
	$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+m^2}} \Rightarrow m^2 + 1 = 2$	
	$\Rightarrow m = 1$ (as $m > 0$)	½

SECTION E		
Q36	In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres. ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM  Based on the above information, answer the following questions : (i) Find the total cost C of digging the tank in terms of x. (ii) Find $\frac{dC}{dx}$. (iii) (a) Find the value of x for which cost C is minimum. **OR** (iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where x > 0.	
Ans(i)	(i)$C = 40000h^2 + 5000x^2$ as $x^2h = 250$ $\Rightarrow C = \frac{40000\ (250)^2}{x^4} + 5000x^2$	 ½ ½
Ans(ii)	(ii) $\frac{dC}{dx} = \frac{-160000\ (250)^2}{x^5} + 10000x$	1
Ans(iii)	(iii)(a)For minimum cost $\frac{dC}{dx} = 0$ $\Rightarrow 10000x^6 = 250 \times 250 \times 160000$ $\Rightarrow x = 10$ showing $\frac{d^2C}{dx^2} > 0$ at $x = 10$ $\therefore$ cost is minimum when $x = 10$	½ 1 ½

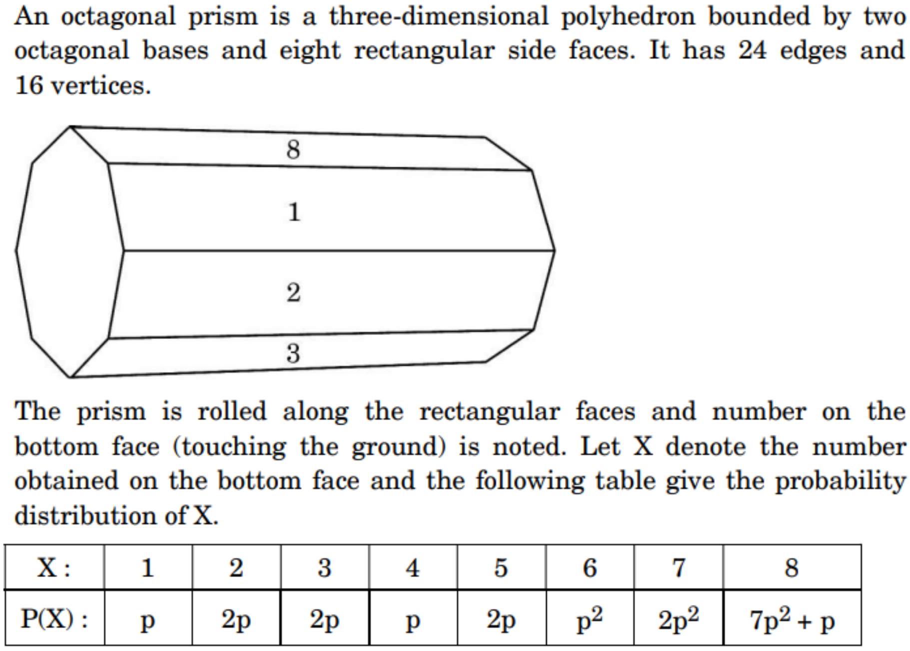
	OR	
Ans(iii)	(iii)(b)$\frac{dC}{dx} = \frac{-160000\ (250)^2}{x^4} + 10000x$	½
	$\frac{dC}{dx} = 0$ gives x = 10	1
	$\frac{dC}{dx} > 0$ in $(10, \infty)$ and $\frac{dC}{dx} < 0$ in (0, 10).	
	Hence, cost function is neither increasing nor decreasing for x > 0	½
Q37		



An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.

The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

X :	1	2	3	4	5	6	7	8
P(X) :	p	2p	2p	p	2p	p^2	$2p^2$	$7p^2 + p$



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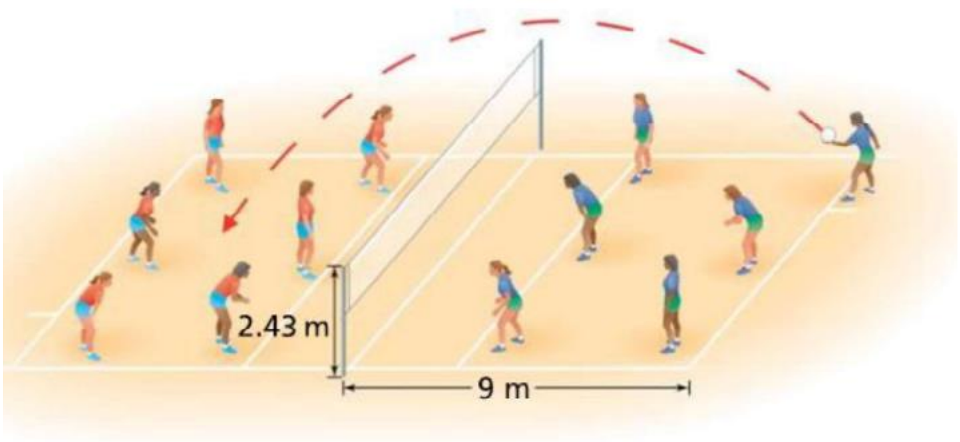
Ans(i)	(i)$10p^2 + 9p = 1$ $\Rightarrow p = \frac{1}{10}$	½ ½
Ans(ii)	(ii)$P(X > 6) = 9p^2 + p$ $= \frac{9}{100} + \frac{1}{10}$ $= \frac{19}{100}$	½ ½
Ans(iii)	(iii)(a)$P(X = 3\,m) = P(3) + P(6)$ $\Rightarrow 2p + p^2 = \frac{21}{100}$	1 1
	OR	
Ans(iii)	(iii)(b) $E(X) = \sum XP(X) = p + 4p + 6p + 4p + 10p + 6p^2 + 14p^2 + 56p^2 + 8p$ $= 33p + 76p^2$ $= \frac{406}{100}$ or $\frac{203}{50}$	1 ½ ½

Q38	A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.  Based on the above information, answer the following questions : (i) Is h(t) a continuous function ? Justify. (ii) Find the time at which the height of the ball is maximum.	
Ans(i)	(i)$h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$ Clearly h(t) is a polynomial function, hence continuous. Hence h(t) is a continuous function.	2
Ans(ii)	(ii)For maximum height , $\frac{dh}{dt} = 0 \Rightarrow -7t + \frac{13}{2} = 0$ $\Rightarrow t = \frac{13}{14}$ $\frac{d^2h}{dt^2} = -7 < 0 \quad \therefore$ height is maximum at $t = \frac{13}{14}$	 1 ½ ½