

**QUESTION PAPER CODE 65/2/2**  
**EXPECTED ANSWER/VALUE POINTS**

**SECTION A**

1.  $|A| = |B| = 0$   $\frac{1}{2}$
- $\Rightarrow |AB| = 0$   $\frac{1}{2}$
2.  $\frac{d}{dx}(e^{\sqrt{3x}}) = \frac{\sqrt{3}}{2\sqrt{x}} e^{\sqrt{3x}}$  1
3. Order = 2, Degree not defined  $\frac{1}{2} + \frac{1}{2}$
4. D. Rs are 1, 1, 1  
 $\therefore$  Direction cosines of the line are:
- $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$  1

OR

Equation of the line is:

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2} \quad 1$$

**SECTION B**

5.  $\int \sqrt{3-2x-x^2} dx = \int \sqrt{2^2 - (x+1)^2} dx$  1
- $$= \frac{x+1}{2} \sqrt{3-2x-x^2} + 2 \sin^{-1} \left( \frac{x+1}{2} \right) + c \quad 1$$
6.  $|A| = p^2 - 4$   $\frac{1}{2}$
- $|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$  1
- $\therefore p^2 - 4 = 5 \Rightarrow p = \pm 3$   $\frac{1}{2}$
7.  $\left. \begin{array}{l} \forall a, b \in \mathbb{R}, \sqrt{a^2 + b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R} \end{array} \right\}$  1

Also,

$$\left. \begin{aligned} a * (b * c) &= a * \sqrt{b^2 + c^2} = \sqrt{a^2 + b^2 + c^2} \\ (a * b) * c &= \sqrt{a^2 + b^2} * c = \sqrt{a^2 + b^2 + c^2} \end{aligned} \right\} \Rightarrow a * (b * c) = (a * b) * c \quad \left. \begin{aligned} & \\ & \therefore * \text{ is Associative} \end{aligned} \right\} \quad 1$$

8. Let  $\theta$  be the angle between  $\vec{a}$  &  $\vec{b}$ , then

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} = \frac{7}{2.7} = \frac{1}{2} \quad 1 \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \quad \frac{1}{2}$$

OR

$$(-3\hat{i} + 7\hat{j} + 5\hat{k}) \cdot \{(-5\hat{i} + 7\hat{j} - 3\hat{k}) \times (7\hat{i} - 5\hat{j} - 3\hat{k})\} = \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix} \quad 1$$

$$= -264 \quad \frac{1}{2}$$

$\therefore$  Volume of cuboid = 264 cubic units

$\frac{1}{2}$

$$9. \text{ (i) } P(3 \text{ heads}) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{5}{16} \quad 1$$

(ii)  $P(\text{At most 3 heads}) = P(r \leq 3)$

$$\left. \begin{aligned} &= 1 - P(4 \text{ heads or } 5 \text{ heads}) \\ &= 1 - \left\{ {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5 \left(\frac{1}{2}\right)^5 \right\} \\ &= \frac{26}{32} \text{ or } \frac{13}{16} \end{aligned} \right\} \quad 1$$

OR

X = No. of heads in simultaneous toss of two coins.

$$X: \quad 0 \quad 1 \quad 2 \quad \frac{1}{2}$$

$$P(x): \quad 1/4 \quad 1/2 \quad 1/4 \quad 1 \frac{1}{2}$$

$$10. \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \cdot \tan x + \operatorname{cosec} x \cdot \cot x) dx \quad 1$$

$$= \sec x - \operatorname{cosec} x + c \quad 1$$

OR

$$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{(x-1)^{-2} - 2(x-1)^{-3}\} dx \quad 1$$

$$\left. \begin{aligned} &= e^x (x-1)^{-2} + c \\ &\text{or} \\ &\frac{e^x}{(x-1)^2} + c \end{aligned} \right\} \quad 1$$

$$11. P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3 \quad \frac{1}{2}$$

$$P(A \cap B) = P(A) \cdot P(B/A) = 0.3 \times 0.5 = 0.15 \quad \frac{1}{2}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70} \text{ or } \frac{3}{14} \quad 1$$

12. Given differential equation can be written as:

$$\frac{dy}{dx} = e^x \cdot e^y \Rightarrow e^{-y} dy = e^x dx \quad 1$$

Integrating both sides, we get

$$-e^{-y} = e^x + c \quad 1$$

## SECTION C

$$13. \sin \left[ \cos^{-1} \left( \frac{4}{5} \right) + \tan^{-1} \left( \frac{2}{3} \right) \right]$$

$$= \sin \left[ \tan^{-1} \left( \frac{3}{4} \right) + \tan^{-1} \left( \frac{2}{3} \right) \right] \quad 1$$

$$= \sin \left[ \tan^{-1} \left( \frac{3/4 + 2/3}{1 - 3/4 \cdot 2/3} \right) \right] = \sin \left[ \tan^{-1} \left( \frac{17}{6} \right) \right] \quad 1 \frac{1}{2}$$

$$= \sin \left[ \sin^{-1} \left( \frac{17}{\sqrt{325}} \right) \right] = \frac{17}{\sqrt{325}} \quad 1 \frac{1}{2}$$

$$\begin{aligned}
 14. \quad & \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} \\
 & = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \left( \begin{array}{l} \text{By applying,} \\ C_1 \rightarrow C_1 + C_2 + C_3 \end{array} \right) & 1 \\
 & = \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \left( \begin{array}{l} \text{By applying,} \\ R_2 \rightarrow R_2 - R_1; \\ R_3 \rightarrow R_3 - R_1 \end{array} \right) & 2 \\
 & = (a+b+c) \{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\} \\
 & = 3(a+b+c) (ab + bc + ca) & 1
 \end{aligned}$$

$$15. \quad R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$$

For  $1 \in A, (1, 1) \notin R \Rightarrow R$  is not reflexive 1

For  $1, 2 \in A, (1, 2) \in R$  but  $(2, 1) \notin R \Rightarrow R$  is not symmetric  $1\frac{1}{2}$

For  $1, 2, 3 \in A, (1, 2), (2, 3) \in R$  but  $(1, 3) \notin R \Rightarrow R$  is not transitive  $1\frac{1}{2}$

OR

**One-One:** Let for  $x_1, x_2 \in N, f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$  2

$\therefore$  'f' is one-one

Onto: co-domain of  $f = \text{Range of } f = Y$  1

$\therefore$  'f' is onto

$\therefore$  f is invertible with,  $f^{-1}: Y \rightarrow N$  and  $f^{-1}(y) = \frac{y-3}{4}$  or  $f^{-1}(x) = \frac{x-3}{4}, x \in Y$  1

$$16. \quad x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x\sqrt{1+y} = -y\sqrt{1+x} \quad \frac{1}{2}$$

Squaring to get:  $x^2(1+y) = y^2(1+x)$   $\frac{1}{2}$

Simplifying to get:  $(x-y)(x+y+xy) = 0$  1

$$\text{As, } x \neq y \quad \therefore y = -\frac{x}{1+x} \quad 1$$

Differentiating w.r.t. 'x', we get:

$$\frac{dy}{dx} = \frac{-1(1+x) - (-x) \cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2} \quad 1$$

OR

$$(\cos x)^y = (\sin y)^x \Rightarrow y \cdot \log (\cos x) = x \cdot \log (\sin y) \quad 1$$

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} \cdot \log (\cos x) + y(-\tan x) = \log(\sin y) + x \cdot \cot y \cdot \frac{dy}{dx} \quad 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \cdot \tan x + \log (\sin y)}{\log (\cos x) - x \cot y} \quad 1$$

$$17. \left. \begin{aligned} \int_0^a f(x) dx &= - \int_a^0 f(a-t) dx \quad \text{Put } x = a - t, dx = -dt \\ & \qquad \qquad \qquad \text{Upper limit} = t = a - x = a - a = 0 \\ & \qquad \qquad \qquad \text{Lower limit} = t = a - x = a - 0 = a \\ &= \int_0^a f(a-t) dt = \int_0^a f(a-x) dx \end{aligned} \right\} \quad 1$$

$$\text{Let } I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx \quad \therefore I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x} dx \quad \dots(ii) \quad 1$$

Adding (i) and (ii) we get

$$\begin{aligned} 2I &= \frac{\pi}{2} \int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx = \frac{\pi}{2\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx \\ &= \frac{\pi}{2\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx \quad \frac{1}{2} \end{aligned}$$

$$\Rightarrow 2I = \frac{\pi}{2\sqrt{2}} \left\{ \log \left| \sec \left( x - \frac{\pi}{4} \right) + \tan \left( x - \frac{\pi}{4} \right) \right| \right\} \Bigg|_0^{\pi/2} \quad \frac{1}{2}$$

$$= \frac{\pi}{2\sqrt{2}} \{ \log |\sqrt{2} + 1| - \log |\sqrt{2} - 1| \} \quad \frac{1}{2}$$

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}} \{ \log (\sqrt{2} + 1) - \log (\sqrt{2} - 1) \} \text{ or } \frac{\pi}{4\sqrt{2}} \log \left( \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right) \quad \frac{1}{2}$$

18.  $\int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} dx = \frac{3}{5} \int \frac{1}{x + 2} dx + \frac{1}{5} \int \frac{2x}{x^2 + 1} dx + \frac{1}{5} \int \frac{1}{x^2 + 1} dx \quad 2$

$$= \frac{3}{5} \log |x + 2| + \frac{1}{5} \log |x^2 + 1| + \frac{1}{5} \tan^{-1} x + c \quad 2$$

19. The given differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan \left( \frac{y}{x} \right) \quad \frac{1}{2}$$

Put  $\frac{y}{x} = v$  and  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ , to get 1

$$v + x \frac{dv}{dx} = v - \tan v \Rightarrow x \frac{dv}{dx} = -\tan v$$

$$\Rightarrow \cot v \, dv = -\frac{1}{x} dx, \quad 1$$

Integrating both sides we get,

$$\left. \begin{aligned} \log |\sin v| &= -\log |x| + \log c \\ \Rightarrow \log |\sin v| &= \log \left| \frac{c}{x} \right| \end{aligned} \right\} \quad 1$$

$\therefore$  Solution of differential equation is

$$\sin \left( \frac{y}{x} \right) = \frac{c}{x} \text{ or } x \cdot \sin \left( \frac{y}{x} \right) = c \quad \frac{1}{2}$$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1 + \sin x} \cdot y = \frac{-x}{1 + \sin x}; \quad \frac{1}{2}$$

$$\text{I.F.} = e^{\int \frac{\cos x}{1 + \sin x} dx} = e^{\log(1 + \sin x)} = 1 + \sin x \quad 1$$

∴ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1 + \sin x} \times (1 + \sin x) dx + c \quad 1$$

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1 + \sin x)} + \frac{c}{(1 + \sin x)} \quad \frac{1}{2}$$

$$20. \quad \vec{a} \cdot \frac{(\vec{b} + \vec{c})}{|\vec{b} + \vec{c}|} = 1 \quad 1$$

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2 + \lambda)^2 + 36 + 4} \quad 1 \frac{1}{2}$$

$$\Rightarrow \lambda + 6 = \sqrt{(2 + \lambda)^2 + 40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1 \quad \frac{1}{2}$$

$$\therefore \text{Unit vector along } (\vec{b} + \vec{c}) \text{ is } \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k} \quad 1$$

$$21. \quad e^{y/x} = \frac{x}{a + bx}, \text{ taking log, on both sides, we get } \frac{y}{x} = \log x - \log(a + bx) \quad 1$$

Differentiating with respect to 'x'

$$\left. \begin{aligned} \frac{x \cdot y' - y}{x^2} &= \frac{1}{x} - \frac{b}{a + bx} = \frac{a}{(a + bx)x} \\ \Rightarrow x \cdot y' - y &= \frac{ax}{a + bx} \quad \dots(i) \end{aligned} \right\} \quad 1 \frac{1}{2}$$

Differentiating with respect to 'x'

$$\Rightarrow x \cdot y'' + y' - y' = \frac{(a + bx) \cdot a - ax \cdot b}{(a + bx)^2} = \left( \frac{a}{a + bx} \right)^2 \quad 1$$

$$\left. \begin{aligned} \Rightarrow x \cdot y'' &= \left( \frac{a}{a + bx} \right)^2 \Rightarrow x^3 \cdot y'' = \left( \frac{ax}{a + bx} \right)^2 \\ &\Rightarrow x^3 \frac{d^2y}{dx^2} = \left\{ x \cdot \frac{dy}{dx} - y \right\}^2 \quad \text{(Using (i))} \end{aligned} \right\} \quad \frac{1}{2}$$

22. Let Edge =  $x$  cm, then

$$V(\text{Volume of cube}) = x^3, S(\text{Surface area}) = 6x^2 \quad 1$$

$$\frac{dv}{dt} = 8 \text{ cm}^3/\text{s} \Rightarrow 3x^2 \frac{dx}{dt} = 8 \Rightarrow \frac{dx}{dt} = \frac{8}{3x^2} \quad 1$$

$$\frac{ds}{dt} = 12x \frac{dx}{dt} = 12x \cdot \frac{8}{3x^2} = \frac{32}{x} \quad 1$$

$$\therefore \left. \frac{ds}{dt} \right|_{x=12} = \frac{32}{12} = \frac{8}{3} \text{ cm}^2/\text{s} \quad 1$$

23. Equation of plane:

$$\Rightarrow \left. \begin{array}{l} \left| \begin{array}{ccc} x-2 & y-5 & z+3 \\ -2-2 & -3-5 & 5+3 \end{array} \right| = 0 \\ \left| \begin{array}{ccc} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \end{array} \right| = 0 \\ \left| \begin{array}{ccc} x-2 & y-5 & z+3 \\ 3 & -2 & 0 \end{array} \right| = 0 \end{array} \right\} \quad 2$$

$$\Rightarrow (x-2)(16) - (y-5)(-24) + (z+3)(32) = 0$$

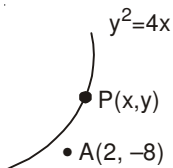
$$\Rightarrow 2x + 3y + 4z - 7 = 0 \quad 1$$

$$\text{Vector form: } \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) - 7 = 0 \quad 1$$

### SECTION D

24.

Let  $P(x, y)$  be any point on the curve  $y^2 = 4x$



$$z = AP = \sqrt{(x-2)^2 + (y+8)^2} \quad 1$$

$$\text{let } s = z^2 = \left( \frac{y^2}{4} - 2 \right)^2 + (y+8)^2 \quad 1$$

$$\frac{ds}{dy} = 2 \left( \frac{y^2}{4} - 2 \right) \left( \frac{y}{2} \right) + 2(y+8) = \frac{y^3}{4} + 16 \quad 1$$

$$\frac{d^2s}{dy^2} = \frac{3y^2}{4} \quad \frac{1}{2}$$

$$\text{Let } \frac{ds}{dy} = 0 \Rightarrow y^3 = -64 \Rightarrow y = -4 \quad 1$$

$$\left. \frac{d^2s}{dy^2} \right|_{y=-4} = \frac{3(16)}{4} > 0 \quad \frac{1}{2}$$

$$\therefore s \text{ or } z \text{ is minimum at } y = -4; x = \frac{y^2}{4} = 4$$

$$\therefore \text{The nearest point is } P(4, -4) \quad 1$$

$$25. \text{ Let } f(x) = x^2 + 2 + e^{2x}, a = 1, b = 3, nh = 2 \quad 1$$

then,

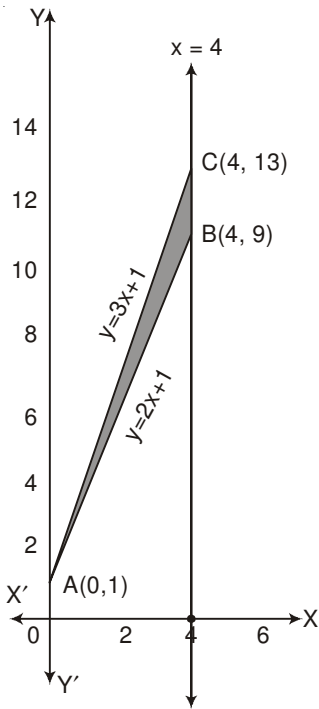
$$\begin{aligned} & f(1) + f(1+h) + f(1+2h) + \dots + f(1 + \overline{n-1}h) \\ &= 3n + h^2[1^2 + 2^2 + \dots + (n-1)^2] + 2h[1 + 2 + \dots + (n-1)] + e^2[1 + e^{2h} + e^{4h} + \dots + e^{2(n-1)h}] \quad 1 \end{aligned}$$

$$= 3n + h^2 \frac{(n-1)(n)(2n-1)}{6} + \frac{2h(n-1)(n)}{2} + \frac{e^2(e^{2nh} - 1)}{e^{2h} - 1} \quad 2$$

$$\int_1^3 (x^2 + 2 + e^{2x}) dx = \lim_{h \rightarrow 0} \left[ 3(nh) + \frac{(nh-h)(nh)(2nh-h)}{6} + (nh)(nh-h) + \frac{e^2}{2} \cdot \frac{2h}{e^{2h}-1} (e^{2nh} - 1) \right] \quad 1$$

$$\left. \begin{aligned} &= 6 + \frac{2 \times 2 \times 4}{6} + 2 \times 2 + \frac{e^2}{2} \times 1 \times (e^4 - 1) \\ &= \frac{38}{3} + \frac{e^2(e^4 - 1)}{2} \end{aligned} \right\} \quad 1$$

OR



Correct figure

1

Points of intersection of given lines

are A(0, 1), B(4, 9), C(4, 13)

$1\frac{1}{2}$

$$\therefore \text{Req. Area} = \int_0^4 (3x + 1) dx - \int_0^4 (2x + 1) dx$$

2

$$= \int_0^4 x dx = \left. \frac{1}{2} x^2 \right|_0^4 = 8$$

$1\frac{1}{2}$

26.  $|A| = 11$ ;  $\text{Adj}(A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$

1+2

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{Adj} A = \frac{1}{11} \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

$\frac{1}{2}$

Taking;  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ;  $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$$A \cdot X = B \quad \therefore X = A^{-1} \cdot B$$

1

$\therefore$  Solution is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

1

$$\therefore x = 1, y = 1, z = 1$$

$\frac{1}{2}$

OR

We know:  $A = IA$ 

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

1

$$R_2 \rightarrow R_2 - 2R_1$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \leftrightarrow R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 + 2R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$$

4

$$R_2 \leftrightarrow R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 3R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

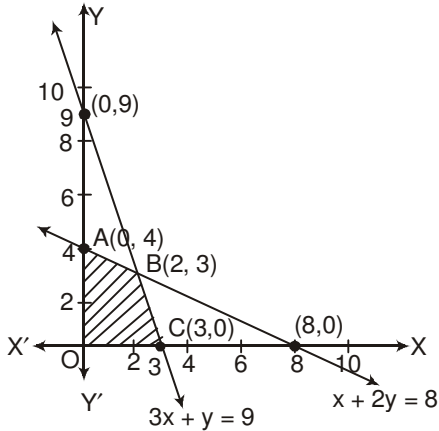
$$R_1 \rightarrow R_1 - R_2 - 2R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

1

27.



let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:

Maximize profit:  $z = 40x + 50y$  (In ₹)

$\frac{1}{2}$

Subject to constraints:

$$\left. \begin{aligned} 3x + y &\leq 9 \\ x + 2y &\leq 8 \\ x, y &\geq 0 \end{aligned} \right\}$$

$2\frac{1}{2}$

Correct graph.

2

Corner point	Value of z (₹)
A(0, 4)	200
B(2, 3)	230 (Max)
C(3, 0)	120

A(0, 4)

200

B(2, 3)

230 (Max)

$\frac{1}{2}$

C(3, 0)

120

$\therefore$  Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units

$\frac{1}{2}$

28. Let  $E_1$  = Event that two-headed coin is chosen  
 $E_2$  = Event that biased coin is chosen  
 $E_3$  = Event that unbiased coin is chosen  
 $A$  = Event that coin tossed shows head

1

Then,  $P(E_1) = P(E_2) = P(E_3) = 1/3$

1

$P(A|E_1) = 1, P(A|E_2) = \frac{75}{100} = \frac{3}{4}, P(A|E_3) = \frac{1}{2}$

$1\frac{1}{2}$

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{4}{9}$$

$2 + \frac{1}{2}$

29. Let  $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ ,  $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$$(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k} \quad 2$$

Vector equation of plane is:

$$\{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$$

$$\Rightarrow \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0 \quad 1$$

Cartesian Equation of plane is:  $-9x - 3y + z + 14 = 0$  1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$$-9(x - 2) - 3(y - 3) + (z - 7) = 0$$

$$\Rightarrow -9x - 3y + z + 20 = 0 \quad 1$$

Distance between parallel planes =  $\left| \frac{-14 + 20}{\sqrt{91}} \right| = \frac{6}{\sqrt{91}}$  1

OR

Equation of line:  $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$  1

Equation of plane:  $\begin{vmatrix} x-2 & y & z-3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$  1

$$\Rightarrow -3(x - 2) + 3y - 3(z - 3) = 0$$

$$\Rightarrow x - y + z - 5 = 0 \quad 1$$

General point on line:  $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$

is:  $P(3k + 2, 4k - 1, 2k + 2)$ ; Putting in the equation of plane 1

we get,  $3k + 2 - 4k + 1 + 2k + 2 = 5 \Rightarrow k = 0$  1

$\therefore$  Point of intersection is: (2, -1, 2) 1