

## Senior School Certificate Examination

March 2019

### Marking Scheme — Mathematics (041) 65/5/1, 65/5/2, 65/5/3

#### **General Instructions:**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
  - Leaving answer or part thereof unassessed in an answer book.
  - Giving more marks for an answer than assigned to it.
  - Wrong transfer of marks from the inside pages of the answer book to the title page.
  - Wrong question wise totaling on the title page.
  - Wrong totaling of marks of the two columns on the title page.
  - Wrong grand total.
  - Marks in words and figures not tallying.
  - Wrong transfer of marks from the answer book to online award list.
  - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
  - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

**QUESTION PAPER CODE 65/5/1**  
**EXPECTED ANSWER/VALUE POINTS**

**SECTION A**

$$1. \sin \theta = \left| \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (3\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{1+1+1} \sqrt{9+1+4}} \right| \quad \frac{1}{2}$$

$$\Rightarrow \theta = \sin^{-1} \left( \frac{4}{\sqrt{42}} \right) \quad \frac{1}{2}$$

OR

$$\text{Any point on } \frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5} = \lambda \dots(1) \quad \frac{1}{2}$$

is  $(\lambda - 2, 3\lambda + 5, 5\lambda - 1)$

Line (1) cuts yz plane at  $\lambda - 2 = 0$  i.e.,  $\lambda = 2$

hence required point is  $(0, 11, 9)$   $\frac{1}{2}$

$$2. y = 5e^{7x} + 6e^{-7x} \Rightarrow \frac{dy}{dx} = 35e^{7x} - 42e^{-7x} \quad \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 245e^{7x} + 294e^{-7x} \Rightarrow \frac{d^2y}{dx^2} = 49y \quad \frac{1}{2}$$

$$3. |2 AA'| = 4|A| |A'| \quad \frac{1}{2}$$

$$= 4 \times 4 \times 4 = 64 \quad \frac{1}{2}$$

$$4. \text{Order} = 2 \quad 1$$

**SECTION B**

$$5. \begin{bmatrix} 2x-3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} \quad 1$$

$$\Rightarrow 2x - 3 = 7 \text{ and } 2y - 4 = 14 \Rightarrow x = 5, y = 9 \Rightarrow x - y = -4 \quad 1$$

$$6. (y + 3x^2) \frac{dx}{dy} = x \Rightarrow \frac{dy}{dx} - \frac{1}{x} y = 3x \quad \frac{1}{2}$$

$$\text{I.F.} = e^{\int \frac{-1}{x} dx} = e^{-\log(x)} = \frac{1}{x} \quad \frac{1}{2}$$

$$\text{Solution is } y \cdot \frac{1}{x} = \int 3x \cdot \frac{1}{x} dx \Rightarrow y = 3x^2 + cx \quad 1$$

$$\begin{aligned} 7. \int e^x (\cos x - \sin x) \operatorname{cosec}^2 x \, dx &= -\int e^x [\operatorname{cosec} x - \operatorname{cosec} x \cot x] \, dx & 1 \\ &= -e^x \operatorname{cosec} x + c & 1 \end{aligned}$$

8. Let P(2, -1, 3), Q(3, -5, 1) and R(-1, 11, 9) be three point.

$$\overline{PQ} = \hat{i} - 4\hat{j} - 2\hat{k} \quad \frac{1}{2}$$

$$\overline{PR} = -3\hat{i} + 12\hat{j} + 6\hat{k} = -3(\hat{i} - 4\hat{j} - 2\hat{k}) \quad \frac{1}{2}$$

$\therefore \overline{PR} = -3\overline{PQ}$ , since P is common.

Therefore the points P, Q and R are collinear. 1

OR

$$(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$$

$$\text{LHS} = (\vec{a} \times \vec{b})^2$$

$$= (|\vec{a}| |\vec{b}| \sin \theta \hat{n})^2 = |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta \quad 1$$

$$= |\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2 \theta)$$

$$= |\vec{a}|^2 |\vec{b}|^2 - (|\vec{a}| |\vec{b}| \cos \theta)^2$$

$$= \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2 \quad 1$$

$$9. \int \frac{x-1}{(x-2)(x-3)} dx = \int \left( \frac{-1}{x-2} + \frac{2}{x-3} \right) dx \quad 1$$

$$= -\log|x-2| + 2\log|x-3| + C \quad 1$$

OR

$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx \quad \text{Put } e^x = t \text{ so that } e^x dx = dt \quad \frac{1}{2}$$

$$\int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{3^2-(t+2)^2}} \quad 1$$

$$= \sin^{-1}\left(\frac{t+2}{3}\right) + C = \sin^{-1}\left(\frac{e^x+2}{3}\right) + C \quad \frac{1}{2}$$

$$10. \quad P(B|A) = 0.4 \Rightarrow \frac{P(B \cap A)}{P(A)} = 0.4 \Rightarrow P(B \cap A) = 0.24 \quad \frac{1}{2}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.5 - 0.24 = 0.86 \quad 1$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.5} = 0.48 \quad \frac{1}{2}$$

$$11. \quad (i) \quad \forall a, b \in \mathbb{Z}, a * b = 2a^2 + b \in \mathbb{Z} \quad \therefore * \text{ is binary} \quad 1$$

$$(ii) \quad 1, 2 \in \mathbb{Z}, 1 * 2 = 2 \times 1^2 + 2 = 4, 2 * 1 = 2 \times (2)^2 + 1 = 9$$

$$1 * 2 \neq 2 * 1$$

$$\therefore * \text{ is not commutative.} \quad 1$$

$$12. \quad n = 4, p = \frac{1}{4}, q = \frac{3}{4} \quad \frac{1}{2}$$

$$P(\text{at least 3 are diamonds}) = P(X = 3) + P(X = 4)$$

$$= {}^4C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right) + {}^4C_4 \left(\frac{1}{4}\right)^4 \quad 1$$

$$= \left(\frac{1}{4}\right)^4 [12 + 1] = \frac{13}{256} \quad \frac{1}{2}$$

OR

Let  $E_1$  : A coming on time.

$E_2$  : B coming on time.

$$P(\bar{E}_1) = \frac{5}{7}, P(\bar{E}_2) = \frac{3}{7} \quad \frac{1}{2}$$

P(only one on time)

$$= P(E_1) P(\bar{E}_2) + P(\bar{E}_1) P(E_2)$$

$$= \frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7}$$

1

$$= \frac{26}{49}$$

 $\frac{1}{2}$ 

13.  $x^p y^q = (x+y)^{p+q} \Rightarrow p \log x + q \log y = (p+q) \log(x+y)$

 $\frac{1}{2}$ 

Differentiating w.r.t x,  $\frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{p+q}{x+y} \left(1 + \frac{dy}{dx}\right)$

1

$$\Rightarrow \left(\frac{q}{y} - \frac{p+q}{x+y}\right) \frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\Rightarrow \left[\frac{qx - py}{y(x+y)}\right] \frac{dy}{dx} = \frac{qx - py}{x(x+y)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

1

Differentiating again w.r.t x

$$\frac{d^2 y}{dx^2} = \frac{x \frac{dy}{dx} - y}{x^2}$$

1

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{x \left(\frac{y}{x}\right) - y}{x^2} \Rightarrow \frac{d^2 y}{dx^2} = 0$$

 $\frac{1}{2}$ 

14.  $\int (\sin x \sin 2x \sin 3x) dx = \frac{1}{2} \int (2 \sin x \sin 2x) \sin 3x dx$

 $\frac{1}{2}$ 

$$= \frac{1}{2} \int [\cos(x) - \cos(3x)] \sin 3x dx$$

1

$$= \frac{1}{4} \int 2 \cos x \sin 3x dx - \frac{1}{4} \int 2 \cos 3x \sin 3x dx$$

$$= \frac{1}{4} \int (\sin 4x + \sin 2x) dx - \frac{1}{4} \int \sin 6x dx$$

1

$$= -\frac{1}{4} \left( \frac{\cos 4x}{4} + \frac{\cos 2x}{2} \right) + \frac{1}{24} \cos 6x + C \quad 1 \frac{1}{2}$$

15. Let  $y = \tan^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right)$ , Put  $x = \tan \theta$  1  $\frac{1}{2}$

$$y = \tan^{-1} \left( \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \Rightarrow y = \tan^{-1} (\tan 3\theta) = 3\theta$$

$$y = 3 \tan^{-1} x \Rightarrow \frac{dy}{dx} = \frac{3}{1+x^2} \quad \dots(i) \quad 1$$

Let  $z = \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$ , put  $x = \sin \phi$  1  $\frac{1}{2}$

$$z = \tan^{-1} \left( \frac{\sin \phi}{\sqrt{1-\sin^2 \phi}} \right) \Rightarrow z = \tan^{-1} (\tan \phi) = \phi$$

$$z = \phi = \sin^{-1} x \Rightarrow \frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(ii) \quad 1$$

Using (i) & (ii),  $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{3\sqrt{1-x^2}}{1+x^2}$  1

OR

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y), \text{ put } x = \sin \theta, y = \sin \phi \quad 1$$

$$\sqrt{1-\sin^2 \theta} + \sqrt{1-\sin^2 \phi} = a(\sin \theta - \sin \phi)$$

$$\Rightarrow \cos \theta + \cos \phi = a(\sin \theta - \sin \phi)$$

$$\Rightarrow 2 \cos \left( \frac{\theta + \phi}{2} \right) \cos \left( \frac{\theta - \phi}{2} \right) = 2a \cos \left( \frac{\theta + \phi}{2} \right) \sin \left( \frac{\theta - \phi}{2} \right) \quad 1$$

$$\Rightarrow \tan \left( \frac{\theta - \phi}{2} \right) = \frac{1}{a}$$

$$\Rightarrow \frac{\theta - \phi}{2} = \tan^{-1} \left( \frac{1}{a} \right) \Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \tan^{-1} \left( \frac{1}{a} \right) \quad 1$$

Differentiating both sides w.r.t x

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0 \quad \frac{1}{2}$$

$$\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \text{ or } \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \quad \frac{1}{2}$$

16.  $(1+e^{2x})dy + (1+y^2)e^x dx = 0$

$$\Rightarrow \int \frac{dy}{1+y^2} = - \int \frac{e^x}{1+e^{2x}} dx \quad 1$$

$$\Rightarrow \tan^{-1} y = - \int \frac{e^x}{1+e^{2x}} dx \quad \frac{1}{2}$$

Put  $e^x = t$ , so that  $e^x dx = dt$   $\frac{1}{2}$

$$\tan^{-1} y = - \int \frac{dt}{1+t^2} \Rightarrow \tan^{-1} y = - \tan^{-1}(e^x) + C \quad \dots(i) \quad 1$$

Substituting  $y = 1$ , when  $x = 0$  in equation (i)

$$\tan^{-1}(1) = - \tan^{-1}(1) + C \Rightarrow C = \frac{\pi}{2} \quad \frac{1}{2}$$

$$\text{Substituting } C = \frac{\pi}{2} \text{ in equation (i)} \Rightarrow \tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2} \quad \frac{1}{2}$$

OR

$$x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{\sin\left(\frac{y}{x}\right)} \quad \dots(i) \quad \frac{1}{2}$$

Put  $\frac{y}{x} = v$  i.e.,  $y = vx$  in (i) so that  $\frac{dy}{dx} = v + x \frac{dv}{dx}$  1

$$v + x \frac{dv}{dx} = v - \frac{1}{\sin v}$$

$$\int \sin v \, dv = \int -\frac{1}{x} \, dx \quad \frac{1}{2}$$

$$\Rightarrow -\cos v = -\log |x| + C \Rightarrow \cos\left(\frac{y}{x}\right) = \log |x| + C \quad \dots(ii) \quad 1$$

Substituting  $y = \frac{\pi}{2}$  when  $x = 1$  in (ii)

$$\cos\left(\frac{\pi}{2}\right) = \log 1 + C \Rightarrow C = 0 \quad \frac{1}{2}$$

$$\text{Required solution is } \cos\left(\frac{y}{x}\right) = \log |x| \quad \frac{1}{2}$$

17. (i) **Reflexive:**  $\forall a \in A, |a - a| = 0$  which is even

$$\Rightarrow (a, a) \in R, \text{ hence } R \text{ is reflexive} \quad 1$$

(ii) **Symmetric:** Let  $(a, b) \in R \Rightarrow |a - b|$  is even

$$\Rightarrow |-(b - a)| \text{ is even} \Rightarrow |b - a| \text{ is even}$$

$$\text{so, } (b, a) \in R$$

$$\text{hence } R \text{ is symmetric.} \quad 1$$

(iii) **Transitive:** Let  $(a, b), (b, c) \in R$

$$\text{so, } |a - b| \text{ is even and } |b - c| \text{ is even}$$

$$\Rightarrow a - b = 2\lambda, b - c = 2\mu \text{ where } \lambda, \mu \in Z$$

$$\text{Now, } a - c = (a - b) + (b - c) = 2(\lambda + \mu)$$

$$\Rightarrow |a - c| \text{ is even, so } (a, c) \in R$$

$$\text{hence } R \text{ is transitive.} \quad 1 \frac{1}{2}$$

$$\text{Since } R \text{ is reflexive, symmetric and transitive therefore its an equivalence relation} \quad \frac{1}{2}$$

OR

$$\text{Let for } x_1, x_2 \in A, f(x_1) = f(x_2) \quad \frac{1}{2}$$

$$\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$$

$$\Rightarrow (4x_1+3)(6x_2-4) = (6x_1-4)(4x_2+3)$$

$$\Rightarrow 34x_1 = 34x_2 \Rightarrow x_1 = x_2, \text{ hence } f \text{ is one-one.}$$

1

For any  $y \in A$  such that  $y = \frac{4x+3}{6x-4}$  there exists  $x$  such that

$$6xy - 4y = 4x + 3 \Rightarrow (6y - 4)x = 4y + 3$$

$$\Rightarrow x = \frac{4y+3}{6y-4}, y \in A, x = \frac{4y+3}{6y-4} \in A$$

1

$\Rightarrow f$  is onto.

1

Since  $f$  is one-one and onto, therefore  $f^{-1}$  exists in  $A$

$$\text{and } f^{-1}(y) = \frac{4y+3}{6y-4} \text{ or } f^{-1}(x) = \frac{4x+3}{6x-4}$$

 $\frac{1}{2}$ 

18.  $f(x) = \cos\left(2x + \frac{\pi}{4}\right) \Rightarrow f'(x) = -2\sin\left(2x + \frac{\pi}{4}\right)$

 $1\frac{1}{2}$ 

$$\text{As } \frac{3\pi}{8} < x < \frac{5\pi}{8} \Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2}$$

 $1\frac{1}{2}$ 

$$\Rightarrow \sin\left(2x + \frac{\pi}{4}\right) < 0 \Rightarrow f'(x) > 0$$

$$\therefore f(x) \text{ is increasing in } \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

1

19. Equation of the required plane is

$$a(x+1) + b(y-3) + c(z-2) = 0 \quad \dots(i)$$

1

Since plane (i) is perpendicular to planes  $x + 2y + 3z = 5$  and  $3x + 3y + z = 0$ , therefore

$$\text{and } \left. \begin{array}{l} a + 2b + 3c = 0 \quad \dots(ii) \\ 3a + 3b + c = 0 \quad \dots(iii) \end{array} \right\}$$

1

Solving (ii) and (iii) we get

$$\frac{a}{7} = \frac{b}{-8} = \frac{c}{3} \quad 1$$

Equation of plane is

$$7 \cdot (x + 1) - 8(y - 3) + 3(z - 2) = 0$$

$$\Rightarrow 7x - 8y + 3z + 25 = 0 \quad 1$$

20. LHS =  $\sin^{-1} \frac{4}{5} + \tan^{-1} \frac{5}{12} + \cos^{-1} \frac{63}{65}$

$$= \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{5}{12} + \cos^{-1} \frac{63}{65} \quad 1$$

$$= \tan^{-1} \left( \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}} \right) + \cos^{-1} \frac{63}{65} \quad 1$$

$$= \tan^{-1} \left( \frac{63}{16} \right) + \cot^{-1} \left( \frac{63}{16} \right) \quad 1$$

$$= \frac{\pi}{2} = \text{RHS} \quad 1$$

21.  $\int_1^5 (|x-1| + |x-2| + |x-4|) dx$

$$= \int_1^2 (5-x) dx + \int_2^4 (x+1) dx + \int_4^5 (3x-7) dx \quad 1 \frac{1}{2}$$

$$= \left[ 5x - \frac{x^2}{2} \right]_1^2 + \left[ \frac{x^2}{2} + x \right]_2^4 + \left[ \frac{3x^2}{2} - 7x \right]_4^5 \quad 1 \frac{1}{2}$$

$$= \frac{7}{2} + 8 + \frac{13}{2} = 18 \quad 1$$

22. A(x, 5, -1), B(3, 2, 1), C(4, 5, 5), D(4, 2, -2)

$$\left. \begin{aligned} \overline{BA} &= (x-3)\hat{i} + 3\hat{j} - 2\hat{k} \\ \overline{BC} &= \hat{i} + 3\hat{j} + 4\hat{k} \\ \overline{BD} &= \hat{i} + 0\hat{j} - 3\hat{k} \end{aligned} \right\} \quad 1 \frac{1}{2}$$

$$\begin{vmatrix} x-3 & 3 & -2 \\ 1 & 3 & 4 \\ 1 & 0 & -3 \end{vmatrix} = 0$$

1

$$\text{i.e., } (x-3)(-9) - 3(-7) - 2(-3) = 0$$

1

$$\Rightarrow x = 6$$

 $\frac{1}{2}$ 23.  $\Delta = 0$ 

$$\Rightarrow \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = 0$$

1

$$\Rightarrow xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

1

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (xyz - 1) = 0$$

 $\frac{1}{2}$ 

$$\Rightarrow \begin{vmatrix} 0 & x-z & x^2-z^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} (xyz-1) = 0 \quad \begin{array}{l} R_1 \rightarrow R_1 - R_3 \\ R_2 \rightarrow R_2 - R_3 \end{array}$$

1

$$\Rightarrow (x-y)(y-z)(z-x)(xyz-1) = 0$$

$$\therefore x \neq y \neq z \text{ therefore } xyz = 1$$

 $\frac{1}{2}$ 

## SECTION D

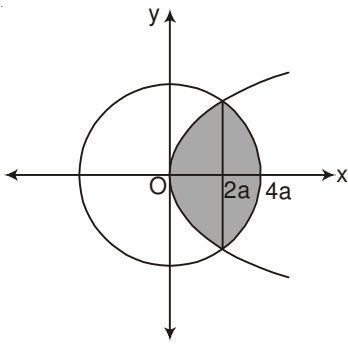
24.

Correct figure

1

$$\text{Solving } x^2 + y^2 = 16a^2 \quad \dots(1)$$

$$\text{and } y^2 = 6ax \quad \dots(2) \text{ we get}$$



$x = 2a$  (as  $-8a$  is not possible)

1

Required Area =  $2 \left[ \int_0^{2a} \sqrt{6ax} \, dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2} \, dx \right]$

2

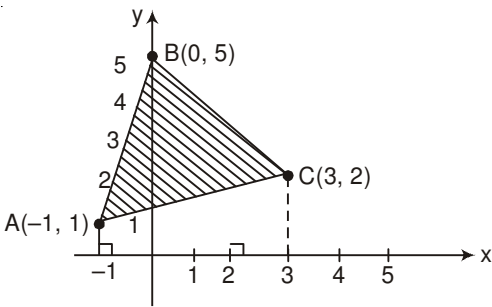
=  $2 \left[ \sqrt{6a} \left( \frac{2}{3} x^{3/2} \right) \right]_0^{2a} + 2 \left[ \frac{x}{2} \sqrt{(4a)^2 - x^2} + 8a^2 \sin^{-1} \left( \frac{x}{4a} \right) \right]_{2a}^{4a}$

1

=  $\frac{8}{3} \sqrt{12} a^2 + \frac{16}{3} \pi a^2 - 4\sqrt{3} a^2$

=  $\frac{4}{3} a^2 (4\pi + \sqrt{3})$

1



OR

Correct figure

1

$4x - y + 5 = 0$

...(1)

$x + y - 5 = 0$

...(2)

$x - 4y + 5 = 0$

...(3)

Coordinates of A(-1, 1), B(0, 5) and C(3, 2)

$1 \frac{1}{2}$

Required Area =  $\int_{-1}^0 (4x + 5) dx + \int_0^3 (5 - x) dx - \frac{1}{4} \int_{-1}^3 (x + 5) dx$

2

=  $\left[ 2x^2 + 5x \right]_{-1}^0 + \left[ 5x - \frac{x^2}{2} \right]_0^3 - \frac{1}{4} \left[ \frac{x^2}{2} + 5x \right]_{-1}^3$

1

=  $\frac{15}{2}$

$\frac{1}{2}$

25. Required equation of the line is

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + \hat{k})$

2

Let  $\vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$ ,  $\vec{a}_2 = \hat{i} + \hat{j}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

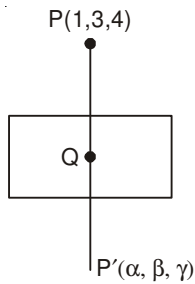
The required distance =  $\left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$

$$= \frac{|(\hat{i} - \hat{k}) \times (2\hat{i} - \hat{j} + \hat{k})|}{|2\hat{i} - \hat{j} + \hat{k}|} \quad 1$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix} = -\hat{i} - 3\hat{j} - \hat{k} \quad 2$$

$$\text{Required distance} = \frac{\sqrt{1+9+1}}{\sqrt{4+1+1}} = \frac{\sqrt{11}}{\sqrt{6}} \text{ or } \frac{\sqrt{66}}{6} \quad 1$$

OR

Correct figure  $\frac{1}{2}$ 

$$\text{Equation of line PQ is } \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \quad 1$$

$$\text{The coordinates of Q are } (2\lambda + 1, -\lambda + 3, \lambda + 4) \quad \frac{1}{2}$$

$$\therefore \text{Q lies on plane } 2x - y + z + 3 = 0$$

$$\therefore 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0 \quad 1$$

$$\Rightarrow 6\lambda + 6 = 0 \text{ i.e., } \lambda = -1 \quad \frac{1}{2}$$

$$\text{The coordinates of Q are } (-1, 4, 3) \quad \frac{1}{2}$$

$$PQ = \sqrt{(-1-1)^2 + (4-3)^2 + (3-4)^2} = \sqrt{6} \quad 1$$

Let  $P'(\alpha, \beta, \gamma)$  be the image of P.

$$\text{then } \frac{\alpha+1}{2} = -1, \frac{\beta+3}{2} = 4, \frac{\gamma+4}{2} = 3 \quad \frac{1}{2}$$

$$\Rightarrow \alpha = -3, \beta = 5, \gamma = 2$$

$$\therefore \text{the image } P' \text{ is } (-3, 5, 2) \quad \frac{1}{2}$$

26. Let numbers of souvenirs of type A be  $x$  and number of souvenirs of type B be  $y$

$\therefore$  L.P.P is

Maximize  $P = 100x + 120y$

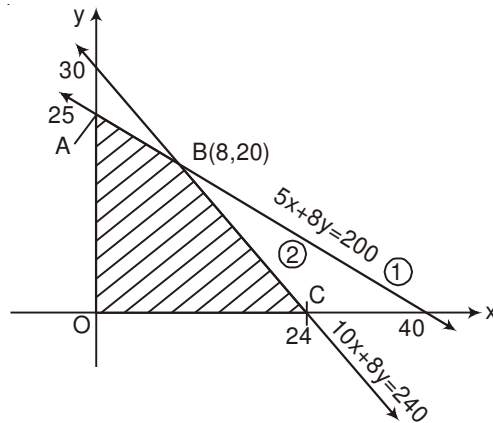
Subject to constraints

$5x + 8y \leq 200$  ... (1)

$10x + 8y \leq 240$  ... (2)

$x, y \geq 0$

$\frac{1}{2}$   
 $2\frac{1}{2}$



2

Values at corner points

Points	P
A(0, 25)	3000
B(8, 20)	3200 (Max)
C(24, 0)	2400

1

So, 8 type A Souvenirs and 20 type B Souvenirs should be made to maximize profit.

27. Let  $E_1 =$  Student guesses the answer  
 $E_2 =$  Student copies the answer  
 $E_3 =$  Student knows the answer  
 $A =$  Student answers the question correctly.

1

$P(E_1) = \frac{1}{4}, P(E_2) = \frac{1}{4}, P(E_3) = 1 - \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2}$

$\frac{1}{2}$

$P(A | E_1) = \frac{1}{4}, P(A | E_2) = \frac{3}{4}, P(A | E_3) = 1$

$\frac{1}{2}$

The required probability

$$= P(E_3 | A) = \frac{P(E_3) \times P(A | E_3)}{\sum_{i=1}^3 P(E_i) \times P(A | E_i)}$$

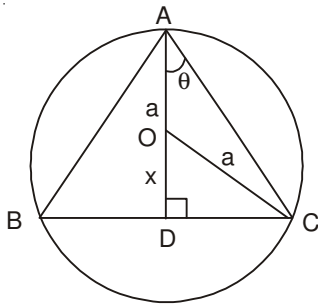
$$= \frac{\frac{1}{2} \times 1}{\frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times 1}$$

$$= \frac{1}{\frac{1}{8} + \frac{3}{8} + 1} = \frac{8}{12} = \frac{2}{3}$$

$\frac{1}{2}$

$\frac{1}{2}$

28.



Correct figure

1

$$CD = \sqrt{a^2 - x^2}$$

$\frac{1}{2}$

$$\text{Area, } A = \frac{1}{2} \times 2\sqrt{a^2 - x^2} (a + x)$$

$\frac{1}{2}$

$$Z = A^2 = (a - x)(a + x)^3$$

$\frac{1}{2}$

$$\frac{dZ}{dx} = 2(a + x)^2 (a - 2x)$$

$\frac{1}{2}$

$$\frac{dZ}{dx} = 0 \Rightarrow x = \frac{a}{2}$$

$\frac{1}{2}$

$$\frac{d^2Z}{dx^2} = -12(a + x)x$$

$\frac{1}{2}$

$$\left( \frac{d^2Z}{dx^2} \right)_{x=\frac{a}{2}} = -9a^2 < 0$$

$\frac{1}{2}$

$\therefore Z$  is maximum when  $x = \frac{a}{2}$

i.e., Area is maximum when  $x = \frac{a}{2}$

$\frac{1}{2}$

For maximum area

$$\tan \theta = \frac{CD}{AD} = \frac{\sqrt{a^2 - \frac{a^2}{4}}}{a + \frac{a}{2}} = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

1

29. Let  $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Then  $A = IA$

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

1

$$\begin{aligned} \Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A & \quad (R_1 \rightarrow R_1 - R_2) \\ \Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A & \quad (R_2 \rightarrow R_2 - 2R_1) \\ \Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 0 & 0 & 1 \end{bmatrix} A & \quad (R_2 \rightarrow R_2 - 2R_3) \\ \Rightarrow \begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A & \quad (R_3 \rightarrow R_3 - 4R_2) \\ \Rightarrow \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} &= \begin{bmatrix} 9 & -13 & 9 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A & \quad (R_1 \rightarrow R_1 - R_3) \\ \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} &= \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} A & \quad (R_1 \rightarrow R_2 - 3R_2) \end{aligned}$$

4

$$\therefore A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \quad 1$$

OR

The given system of equations is

$$AX = B,$$

$$\text{where } A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$|A| = 1200 \neq 0 \quad 1$$

 $\Rightarrow A^{-1}$  exists.

$$X = A^{-1}B \quad 1$$

$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad 2$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad \frac{1}{2}$$

$$X = A^{-1}B = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix} \quad 1$$

$$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5} \quad \frac{1}{2}$$