QUESTION PAPER CODE 65/1/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. order = 2, degree = 1 $\frac{1}{2}+\frac{1}{2}$

2. (fog) (x) = f(x – 7) = x $\frac{1}{2}$

$\Rightarrow \frac{d}{dx}[(fog)(x)] = 1$ $\frac{1}{2}$

3. $\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ $\frac{1}{2}$

$\Rightarrow$ x = 3, y = 3 $\therefore$ x – y = 0 $\frac{1}{2}$

4. d.c.'s = <cos 90°, cos 135°, cos 45°> $\frac{1}{2}$

$= <0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k}) + \lambda(2\hat{i}+2\hat{j}-3\hat{k})$ 1

SECTION B

5. As a, b $\in$ R $\Rightarrow$ ab $\in$ R $\Rightarrow$ ab + 1 $\in$ R $\Rightarrow$ a*b $\in$ R $\Rightarrow$ * is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general (a*b)*c $\neq$ a*(b*c) $\Rightarrow$ * is not associative. 1

6. $A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ 1

$$A^2 - 5A = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} = \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$ 1

7. Let $I = \int \sqrt{1-\sin 2x}\, dx$

$= \int (\sin x - \cos x) dx$ as $\sin x > \cos x$ when $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ 1

$= -\cos x - \sin x + C$ 1

OR

$I = \int \sin^{-1}(2x).1\, dx$

$= x.\sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}} dx$ 1

$= x.\sin^{-1}(2x) + \frac{1}{4}\int \frac{-8x}{\sqrt{1-4x^2}} dx = x\sin^{-1}(2x) + \frac{1}{2}.\sqrt{1-4x^2} + C$ 1

8. $y' = be^{2x} + 2y \Rightarrow b = \frac{y'-2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$\frac{e^{2x}.(y''-2y')-(y'-2y).2x^{2x}}{(e^{2x})^2} = 0$ 1

$\Rightarrow y'' - 4y' + 4y = 0$ or $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ $\frac{1}{2}$

9. Let X: getting an odd number

$p = \frac{1}{2},\ q = \frac{1}{2},\ n = 6$ $\frac{1}{2}$

(i) $P(X=5) = {}^6C_5\left(\frac{1}{2}\right)^6 = \frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X \le 5) = 1 - P(X=6) = 1 - \frac{1}{64} = \frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow \; k = \frac{1}{6}$ 1

10. $A = \{2, 4, 6\}, \;\; B = \{1, 2, 3\}, \;\; A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2}, \; P(B) = \frac{1}{2}, \; P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

11. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow \; 1 + |\vec{a} - \vec{b}|^2 = 2(1+1)$

$\Rightarrow \; |\hat{a} - \hat{b}|^2 = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}$ 1

OR

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$ 1

$= -30$ 1

12. $I = \int \frac{\tan^2 x . \sec^2 x}{1 - (\tan^3 x)^2} dx$

Put $\tan^3 x = t \Rightarrow I = \frac{1}{3} \int \frac{dt}{1 - t^2}$ 1

$= \frac{1}{6} \log \left| \frac{1+t}{1-t} \right| + C = \frac{1}{6} \log \left| \frac{1 + \tan^3 x}{1 - \tan^3 x} \right| + C$ $\frac{1}{2} + \frac{1}{2}$

SECTION C

13. $\tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right)=\frac{\pi}{4}$ 1

$$\Rightarrow \frac{5x}{1-6x^2}=\tan\frac{\pi}{4}=1 \Rightarrow 6x^2+5x-1=0 \qquad 1\frac{1}{2}$$

$$\Rightarrow x=-1 \text{ or } x=\frac{1}{6} \qquad 1$$

as $x=-1$ does not satisfy the given equation,

$\therefore x = \frac{1}{6}$ $\frac{1}{2}$

14. $\log(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)=2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right) \qquad 2$$

$$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right)=\frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right) \qquad 1$$

$$\Rightarrow (x+y)=(x-y)\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{x+y}{x-y} \qquad 1$$

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$$\Rightarrow \frac{du}{dx}-\frac{dv}{dx} = 0 \qquad ...(1) \qquad 1$$

Now, $\log u = y.\log x$

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx}=y\cdot\frac{1}{x}+\log x\cdot\frac{dy}{dx}\Rightarrow\frac{du}{dx}=x^y\left(\frac{y}{x}+\log x\cdot\frac{dy}{dx}\right) \qquad ...(2) \qquad 1$$

Again, log v = x · log y

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y\cdot 1 \Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) \qquad ...(3) \qquad 1$$

From (1), (2) and (3)

$$x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0 \qquad \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x\cdot\log y - x^{y-1}\cdot y}{x^y\cdot\log x - y^{x-1}\cdot x} \qquad \frac{1}{2}$$

15. $$I = \int \frac{3x+5}{x^2+3x-18}\,dx = \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{x^2+3x-18}\,dx \qquad 1$$

$$= \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}\,dx \qquad 1$$

$$= \frac{3}{2}\log|x^2+3x-18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C \qquad 1 + 1$$

16. Let $I = \int_0^a f(a-x)\,dx$

Put a − x = t ⇒ −dx = dt $\qquad \frac{1}{2}$

$$I = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx \qquad \frac{1}{2}$$

II part.

$$I = \int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \qquad I = \int_0^{\pi}\frac{(\pi - x)\cdot\sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi} \frac{\pi \cdot \sin x}{1+\cos^2 x}\, dx$$ $1\frac{1}{2}$

Put cos x = t ⇒ –sinx dx = dt

$$\Rightarrow \ I = -\frac{\pi}{2} \cdot \int_1^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2} \times 2 \times \int_0^1 \frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$ $1\frac{1}{2}$

17. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\sqrt{72}} = -1$ 1

$\Rightarrow \theta = 180°$ or π $\frac{1}{2}$

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

18. LHS = $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$= \begin{vmatrix} a+b+c & a+b & a+c \\ -c & a+b & -(a+c) \\ -b & -(a+b) & (a+c) \end{vmatrix}$$ $1\frac{1}{2}$

$$= (a+b)\,(a+c)\begin{vmatrix} a+b+c & 1 & 1 \\ -c & 1 & -1 \\ -b & -1 & 1 \end{vmatrix}$$ $\frac{1}{2}$

$C_3 \to C_3 + C_2$

$$= (a+b)\,(a+c)\begin{vmatrix} a+b+c & 1 & 2 \\ -c & 1 & 0 \\ -b & -1 & 0 \end{vmatrix}$$ 1

$= 2(a + b)\ (b + c)\ (c + a) = \text{RHS}.$ 1

19. $$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times \left(\sec^2\frac{t}{2} \times \frac{1}{2}\right) = \frac{\cos^2 t}{\sin t}$$ 1

$$\frac{dy}{dt} = \cos t$$ $\frac{1}{2}$

$$\frac{d^2y}{dt^2} = -\sin t \Rightarrow \left.\frac{d^2y}{dt^2}\right]_{t=\frac{\pi}{4}} = -\frac{1}{\sqrt{2}}$$ 1

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$$ $\frac{1}{2}$

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \sec^4 t \cdot \sin t$$

$$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = 2\sqrt{2}$$ 1

20. Clearly $a \le a\ \forall\ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive. $1\frac{1}{2}$

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2,1) \notin R$

$\Rightarrow R$ is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ $\quad (\because x_1, x_2 \in N)$ $\quad 1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $\quad 1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \frac{\sqrt{4x-3}-1}{2}$ $\quad 1$

21. Let the point of contact be $P(x_1, y_1)$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}}$ (slope of tangent)

$\Rightarrow m_1 = \left.\frac{dy}{dx}\right]_{(x_1, y_1)} = \frac{3}{2\sqrt{3x_1-2}}$ $\quad 1$

also, slope of given line = 2 = m_2

$m_1 = m_2 \Rightarrow x_1 = \frac{41}{48}$ $\quad 1$

when $x_1 = \frac{41}{48}, y_1 = \sqrt{\frac{41}{16} - 2} = \frac{3}{4}$ $\quad \therefore P\left(\frac{41}{48}, \frac{3}{4}\right)$ $\quad \frac{1}{2}$

Equation of tangent is: $y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

22. Writing $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$ $\frac{1}{2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

Differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$

$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$ 1

$\Rightarrow \log\left|v + \sqrt{1+v^2}\right| = \log|x| + \log c$ 1

$\Rightarrow v + \sqrt{1+v^2} = cx \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$

when $x = 1, y = 0 \Rightarrow c = 1$ $\frac{1}{2}$

$\therefore y + \sqrt{x^2 + y^2} = x^2$ $\frac{1}{2}$

OR

Given equation is $\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$ $\frac{1}{2}$

I.F. $= e^{\int \frac{2x}{1+x^2}dx} = 1 + x^2$ 1

Solution is given by,

$y \cdot (1 + x^2) = \int \frac{4x^2}{1+x^2} \cdot (1+x^2)\,dx = \int 4x^2\,dx$ 1

$$\Rightarrow \; y \cdot (1+x^2) = \frac{4x^3}{3} + c$$ $\frac{1}{2}$

when $x = 0,\; y = 0 \Rightarrow c = 0$ $\frac{1}{2}$

$$y \cdot (1+x^2) = \frac{4x^3}{3} \;\text{ or }\; y = \frac{4x^3}{3(1+x^2)}$$ $\frac{1}{2}$

23. Given lines are: $\dfrac{x-1}{-3} = \dfrac{y-2}{\left(\frac{\lambda}{7}\right)} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{\left(\frac{-3\lambda}{7}\right)} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$ 1

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{\lambda}{7}\right)(1) + 2(-5) = 0 \Rightarrow \lambda = 7$$ 1

So, lines are

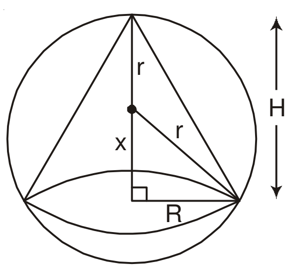
$$\frac{x-1}{-3} = \frac{y-2}{1} = \frac{x-3}{2} \;\text{ and }\; \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$$ $\frac{1}{2}$

$$\text{Consider } \Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = -63$$ 1

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24.



Correct Figure 1

$r^2 = x^2 + R^2$

Now, $V = \dfrac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi(r^2 - x^2)(r + x)$$

$$= \frac{1}{3}\pi(r+x)^2\,(r - x)$$ 1

$$\frac{dV}{dx} = \frac{1}{3}\pi\Big[(r+x)^2\ (-1) + (r-x).2(r+x)\Big]$$

$$= \frac{1}{3}\pi(r+x)\ (r-3x)$$ 1

$$\frac{dV}{dx} = 0 \Rightarrow x = -r \quad \text{or } x = \frac{r}{3}$$ $\frac{1}{2}$

(Rejected)

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi\big[(r+x)(-3) + (r-3x)\big] = -\pi H < 0$$ 1

$\Rightarrow$ V is maximum when $x = \frac{r}{3}$.

$H = r + x = r + \frac{r}{3} = \frac{4r}{3}$ $\frac{1}{2}$

Maximum volume $V = \frac{1}{3}\pi\left(r+\frac{r}{3}\right)^2\left(r-\frac{r}{3}\right) = \frac{32}{81}\pi r^3$ 1

25. $|A| = -1 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$ 2

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Now, $X = A^{-1}B$ 1

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$\Rightarrow$ x = 1, y = 2, z = 3 $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to R_2 + R_1, R_3 \to R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{3}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5/3 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1/3 & 1/3 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 + 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5/3 & -4/3 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{1}{3}R_3, R_2 \to R_2 - \frac{5}{3}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$ 1

26. $\left.\begin{array}{l} \text{Let } E_1 : \text{item is produced by A} \\ \quad E_2 : \text{item is produced by B} \\ \quad E_3 : \text{item is produced by C} \\ \quad A : \text{defective item is found.} \end{array}\right\}$ 1

$P(E_1) = \frac{50}{100}, P(E_2) = \frac{30}{100}, P(E_3) = \frac{20}{100}$ 1

$P(A / E_1) = \frac{1}{100}, P(A / E_2) = \frac{5}{100}, P(A \mid E_3) = \frac{7}{100}$ 1

$$P(E_1 \mid A) = \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$ 2

$$= \frac{5}{34}$$ 1

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

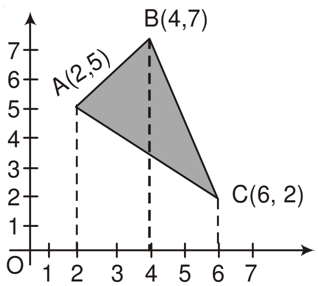
$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3} \qquad 1$$

$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i}+\hat{j}+\hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) = $\dfrac{|-2+1+4|}{\sqrt{(-1)^2+1^2+1^2}} = \sqrt{3}$ 1

28. 

Correct Figure 1

Equation of AB : $y = x + 3$

Equation of BC : $y = \dfrac{-5x}{2} + 17$

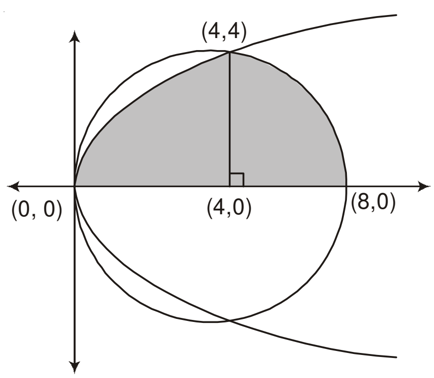
Equation of AC : $y = \dfrac{-3x}{4} + \dfrac{13}{2}$ $1\frac{1}{2}$

Required Area $= \displaystyle\int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2}+17\right)dx - \int_2^6 \left(\frac{-3x}{4}+\frac{13}{2}\right)dx$ $1\frac{1}{2}$

$= \left[\dfrac{(x+3)^2}{2}\right]_2^4 + \left[\dfrac{-5x^2}{4}+17x\right]_4^6 - \left[\dfrac{-3x^2}{8}+\dfrac{13x}{2}\right]_2^6$ $1\frac{1}{2}$

$= 7$ $\frac{1}{2}$

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

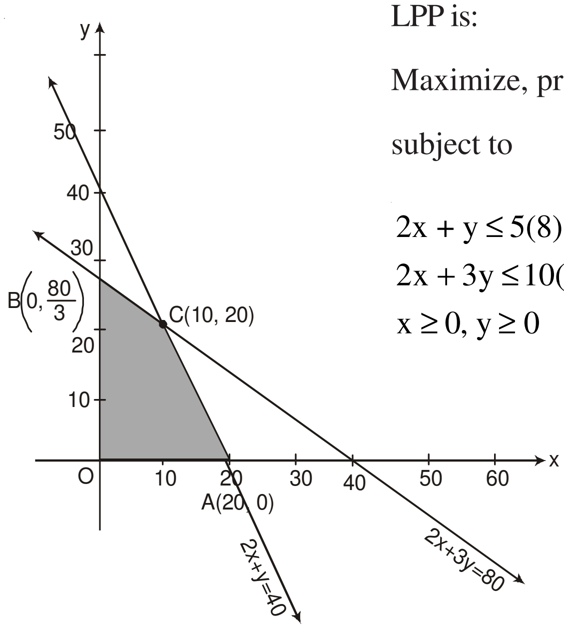
$$\text{Required Area} = \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2-(x-4)^2}\,dx \qquad 1\tfrac{1}{2}$$

$$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8 \qquad 1\tfrac{1}{2}$$

$$= \left(4\pi + \frac{32}{3}\right) \qquad 1$$

Note: A student may also arrive at the answer $\left(8\pi+\frac{64}{3}\right)$ which is double $\left(4\pi+\frac{32}{3}\right)$ because of 'about x-axis'. He/she may be given full marks.

29. Let number of items produced of model A be x and that of model B be y.



LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to

$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 & \end{array}\right\} \qquad 2$$

Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
B$\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10$, $y = 20$. $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.