QUESTION PAPER CODE 430/2/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

Question numbers 1 to 10 are multiple choice questions of 1 mark each. Select the correct option.

1. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?

(a) 2 (b) 4 (c) 5 (d) 1

Sol. (c) 5 1

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2. The maximum number of zeroes a cubic polynomial can have, is

(a) 1 (b) 4 (c) 2 (d) 3

Sol. (d) 3 1

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3. If the centre of a circle is (3, 5) and end points of a diameter are (4, 7) and (2, y), then the value of y is

(a) 3 (b) –3 (c) 7 (d) 4

Sol. (a) 3 1

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4. Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Sol. (d) $\frac{3}{4}$ 1

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5. $2\sqrt{3}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a whole number

Sol. (c) an irrational no. 1

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6. The cumulative frequency table is useful in determining

(a) Mean (b) Median (c) Mode (d) All of these

Sol. (b) Median 1

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7. HCF of two numbers is 27 and their LCM is 162. If one of the number is 54, then the other number is

(a) 36 (b) 35 (c) 9 (d) 81

Sol. (d) 81 1

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8. x-axis divides the line segment joining A(2, –3) and B(5, 6) in the ratio:

(a) 2 : 3 **(b) 3 : 5** **(c) 1 : 2** **(d) 2 : 1**

Sol. (c) 1 : 2 1

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9. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to their product, then k equals.

(a) $\frac{1}{3}$ **(b)** $-\frac{1}{3}$ **(c)** $\frac{2}{3}$ **(d)** $-\frac{2}{3}$

Sol. (d) $-\frac{2}{3}$ 1

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10. A chord of a circle of radius 10 cm, subtends a right angle at its centre. The length of the chord (in cm) is

(a) $\frac{5}{\sqrt{2}}$ **(b)** $5\sqrt{2}$ **(c)** $10\sqrt{2}$ **(d)** $10\sqrt{3}$

Sol. (c) $10\sqrt{2}$ 1

Question numbers 11 to 15, fill in the blanks:

11. The value of ($\tan^2 60° + \sin^2 45°$) is ________.

Sol. $\frac{7}{2}$ or 3.5 1

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12. The corresponding sides of two similar triangles are in the ratio 3 : 4, then the ratios of the area of triangles is _________.

Sol. 9 : 16 1

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13. Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are ________.

Sol. 3 and –2 1

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14. The area of triangle formed with the origin and the points (4, 0) and (0, 6) is ________.

Sol. 12 sq units 1

OR

The co-ordinate of the point dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 is ________.

Sol. (3, 5) 1

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15. **The value of $\frac{\sin\theta}{\cos(90° - \theta)} + \frac{\cos 43°}{\sin 47°}$ is _________**

Sol. 2 1

Question numbers 16 to 20, answer the following :

16. **If 3k – 2, 4k – 6 and k + 2 are three consecutive terms of A.P., then find the value of k.**

Sol. $(4k - 6) - (3k - 2) = (k + 2) - (4k - 6)$ $\frac{1}{2}$

$\Rightarrow k = 3$ $\frac{1}{2}$

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17. **Find the value of (cos 48° – sin 42°).**

Sol. $\cos 48° - \cos (90° - 42°)$ $\frac{1}{2}$

$\cos 48° - \cos 48° = 0$ $\frac{1}{2}$

OR

Evaluate: (tan 23°) × (tan 67°)

Sol. $\tan (90° - 67°) \times \tan 67°$ $\frac{1}{2}$

$= \cot 67° \times \tan 67° = 1$ $\frac{1}{2}$

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18. **In figure-2 $\widehat{PQ}$ and $\widehat{AB}$ are two arcs of concentric circles of radii 7 cm and 3.5 cm resp., with centre O. If ∠POQ = 30°, then find the area of shaded region.**

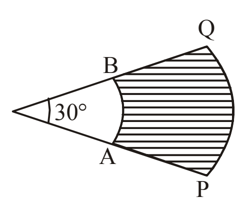

Fig.-2

Sol. Area of shaded region $= \frac{22}{7} \times \frac{30°}{360°}(7^2 - (3.5)^2)$ $\frac{1}{2}$

$= 9.625 \text{ cm}^2$ $\frac{1}{2}$

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19. **A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting a red king.**

Sol. P(Red king) $= \frac{2}{52}$ or $\frac{1}{26}$ 1

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20. Two similar triangles ABC and PQR have their areas 25 cm² and 49 cm² respectively. If QR = 9.8 cm, find BC.

Sol. $\frac{\text{Ar } \Delta ABC}{\text{Ar } \Delta PQR} = \frac{25}{49} \Rightarrow \frac{BC^2}{QR^2} = \frac{25}{49} \Rightarrow \frac{BC}{QR} = \frac{5}{7}$ $\frac{1}{2}$

$BC = \frac{5}{7} \times 9.8 = 7 \text{ cm}$ $\frac{1}{2}$

SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. Divide $(2x^2 - x + 3)$ by $(2 - x)$ and write the quotient and the remainder.

Sol.

$$\begin{array}{r|l} & -2x - 3 \\ -x + 2 & 2x^2 - x + 3 \\ & 2x^2 - 4x \\ & - \quad + \\ \hline & \quad 3x + 3 \\ & \quad 3x - 6 \\ & \quad - \quad + \\ \hline & \quad \quad 9 \end{array}$$ 1

Quotient $= -2x - 3$

$R = 9$ 1

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22. Prove that: $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Sol. $\text{L.H.S} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$ 1

$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$ $\frac{1}{2}$

$= 2\sec^2\theta$ $\frac{1}{2}$

OR

Prove that: $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Sol. $$\text{L.H.S} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$ 1

$$= \cos^2\theta - \sin^2\theta$$ 1

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23. In a family of three children, find the probability of having at least two boys.

Sol. Total outcomes = 8 {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG} 1

P(atleast 2 boys) = $\frac{4}{8}$ or $\frac{1}{2}$ 1

OR

Two dice are tossed simultaneously. Find the probability of getting

(i) an even number on both dice.

(ii) the sum of two numbers more than 9.

Sol. Total outcomes = 36 1

P(even no. on both side) = $\frac{9}{36}$ or $\frac{1}{4}$ $\frac{1}{2}$

P(sum > 9) = $\frac{6}{36}$ or $\frac{1}{6}$ $\frac{1}{2}$

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24. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Sol. Total = 10 + 25 = 35 P(getting prize) = $\frac{10}{35}$ or $\frac{2}{7}$ 1+1

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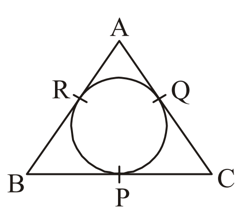
25. An isosceles triangle ABC, with AB = AC, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.

Sol.



AB = AC

AR + RB = AQ + QC $4\times\frac{1}{2}$

RB = QC (AR = AQ)

BP = PC $\Rightarrow$ P bisect BC

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26. The radius of a circle is 17.5 cm. Find the area of the sector of the circle enclosed by two radii and an arc 44 cm in length.

Sol. Area = $\frac{1}{2}lr = \frac{1}{2}\times 44\times 17.5 = 385\ \text{cm}^2$ 1+1

SECTION C

Question numbers 27 to 34 carry 3 marks each.

27. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{p}{q}$ p, q are coprime $q \neq 0$ $\frac{1}{2}$

$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p$ Let $p = 3m$ 1

$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q$ $\frac{1}{2}$

$\therefore$ 3 is common factor of p and q

Contraction to out assumption 1

Hence $\sqrt{3}$ is irrational No.

OR

Using Euclid's algorithm, find the HCF of 272 and 1032.

Sol. $1032 = 272 \times 3 + 216$

$272 = 216 \times 1 + 56$ $\frac{1}{2}+\frac{1}{2}$

$216 = 56 \times 3 + 48$

$56 = 48 \times 1 + 8$ $\frac{1}{2}+\frac{1}{2}$

$48 = 8 \times 6 + 0$ HCF(1032, 272) = 8 $\frac{1}{2}+\frac{1}{2}$

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28. If $x = 3 \sin\theta + 4 \cos\theta$ and $y = 3 \cos\theta - 4 \sin\theta$ then prove that $x^2 + y^2 = 25$.

Sol. $x^2 = 9 \sin^2\theta + 16 \cos^2\theta + 24 \sin\theta \cos\theta$ 1

$y^2 = 9 \cos^2\theta + 16 \sin^2\theta - 24 \sin\theta \cos\theta$ 1

$x^2 + y^2 = 25$ 1

OR

If $\sin\theta + \sin^2\theta = 1$; then prove that $\cos^2\theta + \cos^4\theta = 1$.

Sol. $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$ 1

L.H.S $= \cos^2\theta + (\cos^2\theta)^2 = \cos^2\theta + \sin^2\theta$ 1+1

$= 1 =$ R.H.S

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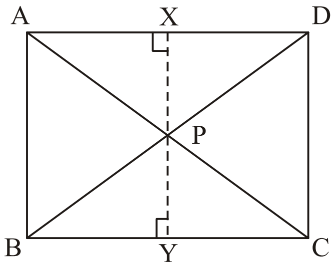
29. In a rectangle ABCD, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Sol.



Correct figure & Construction $\frac{1}{2}+\frac{1}{2}$

In rt ΔAPX $AP^2 = AX^2 + PX^2$

In rt ΔPCY $PC^2 = PY^2 + YC^2$ $\frac{1}{2}$

In rt ΔPBY $PB^2 = PY^2 + BY^2$

In rt ΔPXD $PD^2 = DX^2 + PX^2$ $\frac{1}{2}$

$PA^2 + PC^2 = AX^2 + PX^2 + PY^2 + YC^2$

$= BY^2 + PY^2 + PX^2 + XD^2$ 1

$= PB^2 + PD^2$

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30. Draw a line segment of length 7 cm and divide it in the ratio 2 : 3.

Sol. Correct construction 3

OR

Draw a circle of radius 4 cm and construct the pair of tangents to the circle from an external point, which is at a distance of 7 cm from its centre.

Sol. Correct construction 3

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31. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 3. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

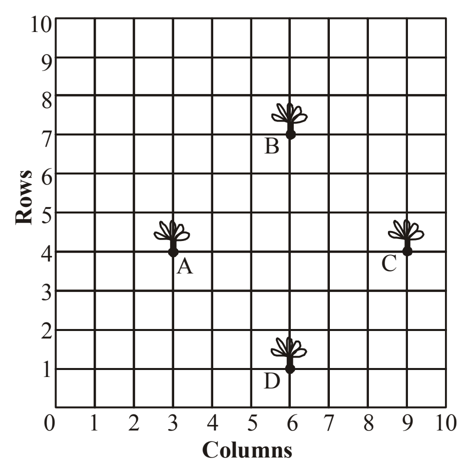

Fig. 3

Sol. A = (3, 4), B = (6, 7), C = (9, 4), D = (6, 1) 1

$AB = 3\sqrt{2}, \quad BC = 3\sqrt{2}, \quad CD = 3\sqrt{2}, \quad DA = 3\sqrt{2}$ 1

AC = 6 unit BD = 6 unit $\frac{1}{2}$

AB = BC = CD = DA and AC = BD

ABCD is a square

∴ Champa is correct $\frac{1}{2}$

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32. Solve graphically:

2x – 3y + 13 = 0; 3x – 2y + 12 = 0

Sol. Correct graph of 2x – 3y + 13 = 0, 3x – 2y + 12 = 0 1+1

Solution x = –2, y = 3 1

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33. A horse is tethered to one corner of a rectangular field of dimensions 70 m × 52 m, by a rope of length 21 m. How much area of the field can it graze?

Sol. Area of field = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$ 1+1

$= 346.5 \text{ cm}^2$ 1

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34. Find the quadratic polynomial, the sum and product of whose zeroes are –3 and 2 respectively. Hence find the zeroes.

Sol. Polynomial $K(x^2 + 3x + 2)$

Put K = 1 ⇒ required polynomial $x^2 + 3x + 2$ $1\frac{1}{2}$

$x^2 + 3x + 2 = (x + 2)(x + 1)$ 1

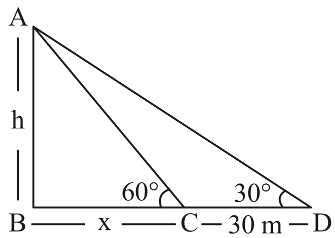
∴ Zeroes are –2, –1 $\frac{1}{2}$

SECTION D

Question numbers 35 to 40 carry 4 marks each.

35. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and width of the river. [Take $\sqrt{3}$ = 1.732]

Sol.



Correct figure 1

In right ΔABC

$\tan 60° = \frac{h}{x}$ $\frac{1}{2}$

$\sqrt{3}x = h$ ...(1) $\frac{1}{2}$

In rt ΔABD $\tan 30° = \frac{h}{30+x} \Rightarrow \frac{30+x}{\sqrt{3}} = h$ ...(2) $\frac{1}{2}+\frac{1}{2}$

Solving (1) & (2) $x = 15\text{m}$, $h = 15\sqrt{3}\text{ m} = 25.98\text{ m}$ $\frac{1}{2}+\frac{1}{2}$

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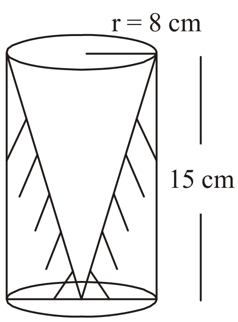
36. From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of remaining solid. (Give your answer in terms of π).

Sol. Correct figure $\frac{1}{2}$



$l = 17$ 1

$r = 8$ cm $\frac{1}{2}$

Total S.A. of remaining solid= C.S.A of cylinder + C.S.A of cone + Area of base

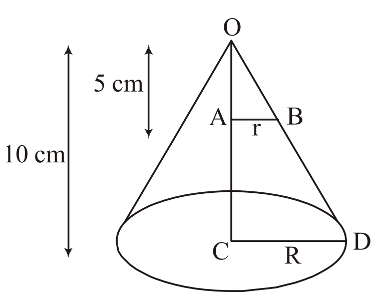
$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$ 1

$= \pi \times 8(2 \times 15 + 17 + 8) = 8\pi(55) = 440\pi \text{ cm}^2$ 1

OR

The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.

Sol. For correct fig 1



ΔOAB ~ ΔOCD

$\frac{OA}{OC} = \frac{AB}{CD} \Rightarrow \frac{5}{10} = \frac{r}{R}$

$\Rightarrow R = 2r$ 1

$\frac{\text{V of cone}}{\text{V of frustum}} = \frac{\frac{1}{3}\pi r^2 5}{\frac{1}{3}\pi(r^2 + R^2 + rR)} = \frac{r^2}{7r^2} = \frac{1}{7}$ 1+1

or 7 : 1

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37. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Sol. Correct Fig., given, to prove, construction $4\times\frac{1}{2}=2$

Correct proof given, to prove, construction, 2

OR

Prove that the length of tangents drawn from an external point to a circle are equal.

Sol. Correct Fig., given, to prove, construction $4\times\frac{1}{2}=2$

Correct proof given, to prove, construction, 2

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38. The 17^{th} term of an A.P. is 5 more than twice its 8th term. If 11th term of A.P. is 43; then find its nth term.

Sol. $a_{17} = 2a_8 + 5 \Rightarrow a + 16d = 2(a + 7d) + 5$ 1

$\Rightarrow 2d - a = 15$ ...(1)

$a_{11} = 43 \Rightarrow a + 10d = 43$ ...(2) 1

Solving (1) & (2) a = 3 d = 4 1

$a_n = 4n - 1$ 1

OR

How many terms of A.P. 3, 5, 7, 9, ... must be taken to get the sum 120?

Sol. a = 3, d = 3, Sn = 120 1

$\frac{n}{2}[2\times 3+(n-1)2]=120 \Rightarrow n^2 + 2n - 120 = 0$ 1

$(n + 12)(n - 10) = 0$ 1

$n = -12, n = 10$ 1

Reject $n = -12$, $n = 10$

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39. Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46. Find the integers.

Sol. Let three consecutive +ve integers x, x + 1, x + 2

$x^2 + (x + 1)(x + 2) = 46$ 1

$2x^2 + 3x - 44 = 0 \Rightarrow 2x^2 + 11x - 8x - 44 = 0$ 1

$\Rightarrow (2x + 11)(x - 4) = 0$

$\Rightarrow \ x = \frac{-11}{2}, x = 4$ 1

$\Rightarrow$ 3 consecutive integers are 4, 5, 6 1

40. Find the mean of the following distribution:

Class	**10 – 25**	**25 – 40**	**40 – 55**	**55 – 70**	**70 – 85**	**85 – 100**
Frequency	**2**	**3**	**7**	**6**	**6**	**6**

Sol.

C.I.	x_i	**f**	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
10-25	17.5	2	–2	–4
25-40	32.5	3	–1	–3
40-55	47.5 a	7	0	0
55-70	62.5	6	1	6
70-85	77.5	6	2	12
85-100	92.5	6	3	18
		30		29

Correct Table 2

Mean $= 47.5 + \frac{29}{30} \times 15$ 1

$= 47.5 + 14.5 = 62$ 1