QUESTION PAPER CODE 30/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Let the point A be (x, y)

$\therefore \quad \frac{x+3}{2} = -2$ and $\frac{y+4}{2} = 2$ $\frac{1}{2}$

$\Rightarrow \quad x = -7$ and $y = 0$

Point is (–7, 0) $\frac{1}{2}$

2. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.) 1

e.g., 1.5, 1.6, 1.63 etc.

3. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore \quad 99 = 12 + (n - 1) \times 3$

$\Rightarrow \quad n = 30$ $\frac{1}{2}$

4. tan 2 A = cot (90° – 2A)

$\therefore \quad 90° - 2A = A - 24°$ $\frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\frac{1}{2}$

5. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ $\frac{1}{2}$

$\Rightarrow \quad k \leq 4$ $\frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots = 1 $\frac{1}{2}$

$\Rightarrow \frac{k}{3} = 1 \Rightarrow k = 3$ $\frac{1}{2}$

6. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore \frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore ar(\Delta ABC) : ar(\Delta ADE) = 9 : 1$

SECTION B

7. System of equations has infinitely many solutions.

$\therefore \frac{2}{k+1} = \frac{3}{2k-1} = \frac{7}{4k+1}$ $\frac{1}{2}$

$\Rightarrow 4k - 2 = 3k + 3$ $\frac{1}{2}$

$\Rightarrow k = 5$

Also $12k + 3 = 14k - 7$ $\frac{1}{2}$

$\Rightarrow k = 5$

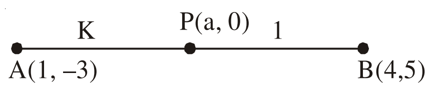
Hence k = 5. $\frac{1}{2}$

8. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$. 1

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$\therefore \quad a = \frac{4k+1}{k+1}$

$0 = \frac{5k-3}{k+1}$

$\Rightarrow \quad k = \frac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\frac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) = $1-\frac{1}{4}=\frac{3}{4}$. $\frac{1}{2}$

11. $a_n = a_{21} + 120$

$= (3 + 20 \times 12) + 120$

$= 363$ 1

$\therefore \quad 363 = 3 + (n - 1) \times 12$

$\Rightarrow \quad n = 31$ 1

or 31st term is 120 more than a_{21}.

OR

$a_1 = S_1 = 3 - 4 = -1$ $\frac{1}{2}$

$a_2 = S_2 - S_1 = [3(2)^2 - 4(2)] - (-1) = 5$ $\frac{1}{2}$

$\therefore d = a_2 - a_1 = 6$ $\frac{1}{2}$

Hence $a_n = -1 + (n - 1) \times 6 = 6n - 7$ $\frac{1}{2}$

Alternate method:

$S_n = 3n^2 - 4n$

$\therefore \quad S_{n-1} = 3(n-1)^2 - 4(n-1) = 3n^2 - 10n + 7$ 1

Hence $a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - (3n^2 - 10n + 7)$ $\frac{1}{2}$

$= 6n - 7$ $\frac{1}{2}$

12. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

$a = 4q + r, \; 0 \le r < 4$

$\Rightarrow \quad a = 4q, \; a = 4q + 1, \; a = 4q + 2$ and $a = 4q + 3$. 1

Now $a = 4q$ and $a = 4q + 2$ are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. $\frac{1}{2}$

SECTION C

13.

Class	x	Freq (f)	$u = \frac{x-50}{20}$	fu
0-20	10	12	–2	–24
20-40	30	15	–1	–15
40-60	50	32	0	0
60-80	70	k	1	k
80-100	90	13	2	26
		72 + k		–13 + k

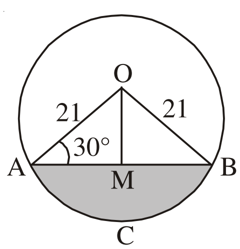
Correct Table 2

$\overline{x} = 53 = 50 + 20 \times \frac{-13+k}{72+k}$

$\Rightarrow$ 3k + 216 = 20k – 260

$\Rightarrow$ k = 28 1

14. Draw OM $\perp$ AB



$\angle OAB = \angle OBA = 30°$ $\frac{1}{2}$

$\sin 30° = \frac{1}{2} = \frac{OM}{21} \Rightarrow OM = \frac{21}{2}$

$\cos 30° = \frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow AM = \frac{21}{2}\sqrt{3}$

Area of $\Delta OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

$= \frac{441}{4}\sqrt{3}\ cm^2.$ 1

$\therefore$ Area of shaded region = Area (sector OACB) – Area (ΔOAB)

$= \frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$ 1

$= \left(462 - 441\frac{\sqrt{3}}{4}\right) cm^2$ or $271.3\ cm^2$ (approx.) $\frac{1}{2}$

15. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{CD}$ ...(1) 1

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

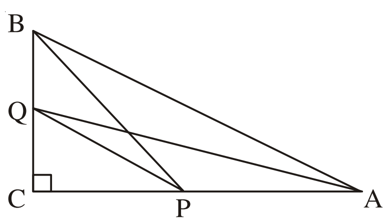
$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$\frac{AD}{CD} = \frac{CD}{BD}$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

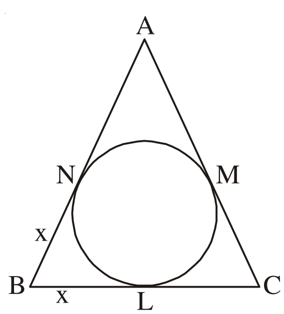
$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore\ AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2$. 1

16.



Let BL = x = BN

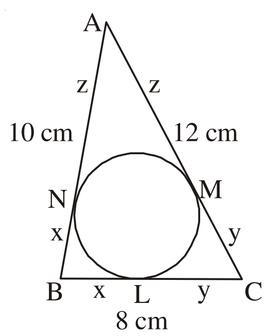
$\left.\begin{array}{l}\therefore\ CL = 8 - x = CM \\ \because\ AC = 12 \Rightarrow AM = 4 + x = AN\end{array}\right\}$ 1

Now AB = AN + NB = 10 $\Rightarrow$ x + 4 + x = 10

$\Rightarrow$ x = 3 1

$\therefore$ BL = 3 cm, CM = 5 cm and AN = 7 cm 1

Alternate method



Let BL = BN = x (tangents from external points are equal) $\frac{1}{2}$

CL = CM = y

AN = AM = z

$\therefore$ AB + BC + AC = 2x + 2y + 2z = 30

$\Rightarrow$ x + y + z = 15 ...(i) 1

Also x + z = 10, x + y = 8 and y + z = 12

Subtracting from equation (i)

y = 5, z = 7 and x = 3 $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

$\therefore$ BL = 3 cm, CM = 5 cm and AN = 7 cm.

17. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000 \text{ m}^3$ 1

If 8 cm standing water is needed

then area irrigated = $\frac{6 \times 1.5 \times 5000}{.08} = 562500 \text{ m}^2$. $1+\frac{1}{2}$

18. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and b ≠ 0.

So $\sqrt{2} = \frac{a}{b}$

$\Rightarrow$ $a^2 = 2b^2$ 1

Thus a^2 is a multiple of 2

$\Rightarrow$ a is a multiple of 2. $\frac{1}{2}$

Let a = 2 m for some integer m

$\therefore$ $b^2 = 2m^2$ $\frac{1}{2}$

Thus b^2 is a multiple of 2

$\Rightarrow$ b is a multiple of 2 $\frac{1}{2}$

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number. $\frac{1}{2}$

19. Sum of zeroes = k + 6 1

Product of zeroes = 2(2k – 1) 1

Hence $k + 6 = \frac{1}{2} \times 2(2k - 1)$

$\Rightarrow \quad k = 7$ 1

20. Let the required point on y-axis be (0, b) $\frac{1}{2}$

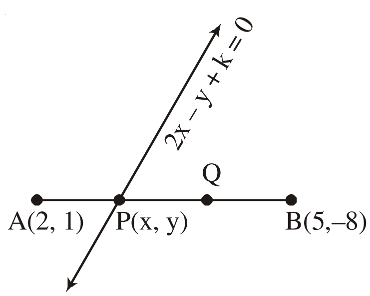
$\therefore \quad (5 - 0)^2 + (-2 - b)^2 = (-3 - 0)^2 + (2 - b)^2$ 1

$\Rightarrow \quad 29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow \quad b = -2$ 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2} + \frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on 2x – y + k = 0

$\Rightarrow \quad 6 + 2 + k = 0$

$\Rightarrow \quad k = -8.$ 1

21. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore \quad y = 3x$ ...(1) 1

and $y + 5 = 2(x + 10)$ ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get x = 7, y = 15.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

22. LHS $= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta = \text{RHS}$ $\frac{1}{2}$

OR

LHS $= \left(1 + \frac{1}{\tan A} - \text{cosec}\,A\right)(1 + \tan A + \sec A)$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)(1 + \tan A + \sec A)$ 1

$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 = \text{RHS}$ 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$ 1

$$= (\sin A + \cos A - 1)(\cos A + \sin A + 1)\cdot\frac{1}{\cos A \sin A}$$

$$= \left[(\sin A + \cos A)^2 - 1\right]\times\frac{1}{\sin A \cos A}$$ 1

$$= (1 + 2\sin A \cos - 1)\times\frac{1}{\sin A \cos A}$$ $\frac{1}{2}$

$$= 2 = \text{RHS}$$ $\frac{1}{2}$

SECTION D

23. $$\text{LHS} = \frac{\sin^2 A/\cos^2 A}{\frac{\sin^2 A}{\cos^2 A}-1}+\frac{1/\sin^2 A}{\frac{1}{\cos^2 A}-\frac{1}{\sin^2 A}}$$ 1

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A}+\frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$ 1

$$= \frac{1}{\sin^2 A - \cos^2 A}$$ 1

$$= \frac{1}{1-2\cos^2 A}$$ 1

24. Here $a = 3$, $a_n = 83$ and $S_n = 903$

Therefore $83 = 3 + (n - 1)d$

$\Rightarrow$ $(n - 1)d = 80$ ...(i) 1

Also $903 = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(6+80) = 43n$ (using (i)) $1+\frac{1}{2}$

$\Rightarrow$ $n = 21$

and $d = 4$ $1\frac{1}{2}$

25. Correct construction of ΔABC 2

Correct construction of triangle similar to ΔABC. 2

26.

Class	Frequency	Cumulative freq.		
0-10	f_1	f_1		
10-20	5	$5 + f_1$		
20-30	9	$14 + f_1$		
30-40	12	$26 + f_1$		
40-50	f_2	$26 + f_1 + f_2$		
50-60	3	$29 + f_1 + f_2$		
60-70	2	$31 + f_1 + f_2$	Correct Table	1
	40			

Median = 32.5 ⇒ median class is 30-40. $\frac{1}{2}$

Now $32.5 = 30 + \frac{10}{12}(20 - 14 - f_1)$ 1

$\Rightarrow f_1 = 3$ 1

Also $31 + f_1 + f_2 = 40$

$\Rightarrow f_2 = 6$ $\frac{1}{2}$

OR

Less than type distribution is as follows

Marks	No. of students		
Less than 5	2		
Less than 10	7		
Less than 15	13		
Less than 20	21		
Less than 25	31		
Less than 30	56		
Less than 35	76		
Less than 40	94		
Less than 45	98		
Less than 50	100	Correct Table	$1\frac{1}{2}$

Plotting of points (5, 2), (10, 7) (15, 13), (20, 21), (25, 31), (30, 56),

(35, 76), (40, 94), (45, 98), (50, 100) $1\frac{1}{2}$

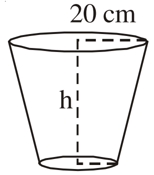
Joining to get the curve $\frac{1}{2}$

Getting median from graph (approx. 29) $\frac{1}{2}$

27. Correct given, to prove, figure and construction $\frac{1}{2}\times 4=2$

Correct proof. 2

28.



Volume of the bucket = 12308.8 cm^3

Let r_1 = 20 cm, r_2 = 12 cm

$\therefore \quad V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \frac{3.14 \times h}{3}(400 + 144 + 240)$ 1

$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784} = 15$ cm 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l\,(r_1 + r_2)$

$= 3.14\,(144 + 17 \times 32)$

$= 2160.32$ cm^2. 1

29. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ 2

$\Rightarrow \quad 4x^2 - 23x + 15 = 0$ $\frac{1}{2}$

$\Rightarrow \quad (4x - 3)(x - 5) = 0$

$x \neq \frac{3}{4} \quad \therefore\ x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\frac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ...(i) 1

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

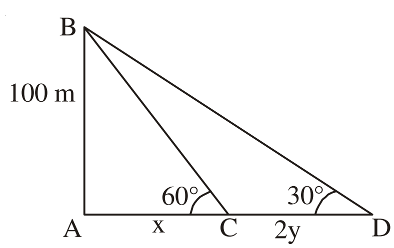
$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get x = 8, y = 3. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

30. Correct Figure 1



Let the speed of the boat be y m/min

$\therefore\ CD = 2y$

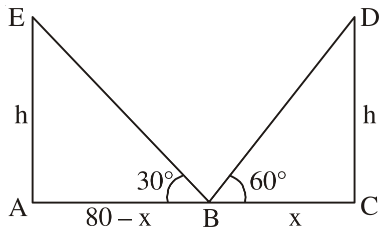
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore\ y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR



Correct Figure 1

Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80-x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1