Senior School Certificate Examination-2020
Marking Scheme - MATHEMATICS
Subject Code: 041 Paper Code: 65/3/1

***General instructions*:-**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X"be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
5. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
6. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
7. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
8. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
9. A full scale of marks 0 - 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
10. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
11. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
12. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
13. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
14. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
15. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
16. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/3/1

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions. Select the correct option.

Q.No. **Marks**

1. If f and g are two functions from R to R defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then $f \circ g(x)$ for $x < 0$ is

(A) $4x$ **(B)** $2x$ **(C)** 0 **(D)** $-4x$

Ans: (D) $-4x$ 1

2. The principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is

(A) $-\frac{\pi}{6}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{2\pi}{3}$ **(D)** $\frac{5\pi}{6}$

Ans: (D) $\frac{5\pi}{6}$ 1

3. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

(A) 64 **(B)** 16 **(C)** 0 **(D)** -8

Ans: (A) 64 1

4. The maximum value of slope of the curve $y = -x^3 + 3x^2 + 12x - 5$ is

(A) 15 **(B)** 12 **(C)** 9 **(D)** 0

Ans: (A) 15 1

5. $\int \frac{e^x(1+x)}{\cos^2\left(xe^x\right)}dx$ is equal to

(A) $\tan\left(xe^x\right)+c$ **(B)** $\cot\left(xe^x\right)+c$

(C) $\cot\left(e^x\right)+c$ **(D)** $\tan\left[e^x(1+x)\right]+c$

Ans: (A) $\tan\left(xe^x\right)+c$ 1

6. The degree of the differential equation $x^2\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6

Ans: (A) 1 1

7. The value of p for which $p(\hat{i}+\hat{j}+\hat{k})$ is a unit vector is

(A) 0 **(B)** $\frac{1}{\sqrt{3}}$ **(C)** 1 **(D)** $\sqrt{3}$

Ans: (B) $\frac{1}{\sqrt{3}}$ **1**

8. The coordinates of the foot of the perpendicular drawn from the point (–2, 8, 7) on the XZ-plane is

(A) (–2, –8, 7) **(B)** (2, 8, –7) **(C)** (–2, 0, 7) **(D)** (0, 8, 0)

Ans: (C) (–2, 0, 7) **1**

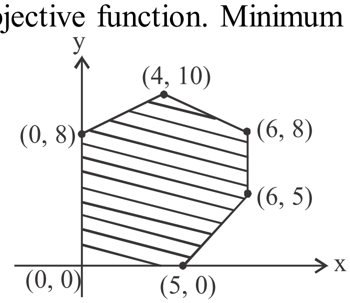
9. The vector equation of XY-plane is

(A) $\vec{r}\cdot\hat{k}=0$ **(B)** $\vec{r}\cdot\hat{j}=0$ **(C)** $\vec{r}\cdot\hat{i}=0$ **(D)** $\vec{r}\cdot\vec{n}=1$

Ans: (A) $\vec{r}\cdot\hat{k}=0$ **1**

10. The feasible region for an LPP is shown below:

Let z = 3x– 4y be the objective function. Minimum of z occurs at



(A) (0, 0) **(B)** (0, 8) **(C)** (5, 0) **(D)** (4, 10)

Ans: (B) (0, 8) **1**

Fill in the blanks in questions numbers 11 to 15

11. If $y = \tan^{-1} x + \cot^{-1} x$, $x \in R$, then $\frac{dy}{dx}$ is equal to __________.

Ans: 0 **1**

OR

If $\cos(xy) = k$, where k is a constant and $xy \neq n\pi$, $n \in Z$,

then $\frac{dy}{dx}$ is equal to ____________.

Ans: $-\frac{y}{x}$ **1**

12. The value of λ so that the function f defined by $f(x) = \begin{cases} \lambda x, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases}$

is continuous at $x = \pi$ is ______

Ans: $-\frac{1}{\pi}$ **1**

13. The equation of the tangent to the curve $y = \sec x$ at the point (0, 1) is __________.

Ans: $y = 1$ **1**

14. The area of the parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is

__________ square units.

Ans: 3 **1**

OR

The value of λ for which the vectors $2\hat{i} - \lambda\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$

are orthogonal is ________.

Ans: $\frac{1}{2}$ **1**

15. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is ________

Ans: $\frac{2}{7}$ **1**

Question numbers 16 to 20 are very short answer type questions

16. Construct a 2 × 2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |(i)^2 - j|$.

Ans: $\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$ $\frac{1}{2}$ mark for any two correct = **1**

17. Differentiate $\sin^2\left(\sqrt{x}\right)$ with respect to x.

Ans: $\frac{\sin\left(2\sqrt{x}\right)}{2\sqrt{x}}$ or $\frac{\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$ **1**

18. Find the interval in which the function f given by $f(x) = 7 - 4x - x^2$ is strictly increasing.

Ans: $f'(x) = -4 - 2x$ **1/2**

$\Rightarrow$ f(x) is increasing on $(-\infty, -2)$ **1/2**

19. Evaluate: $\int_{-2}^{2} |x|\, dx$.

Ans: $\int_{-2}^{2} |x| dx = -\int_{-2}^{0} x dx + \int_{0}^{2} x dx = 4$ **1/2+1/2**

OR

Find $\int \frac{dx}{9+4x^2}$

Ans: $\int \frac{dx}{9+4x^2} = \frac{1}{6}\tan^{-1}\frac{2x}{3} + c$ **1/2+1/2**

20. An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

Ans: $1-\left(\frac{1}{2}\right)^4 = \frac{15}{16}$ **1/2+1/2**

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. Solve for x: $\sin^{-1} 4x + \sin^{-1} 3x = -\frac{\pi}{2}$

Ans: $\sin^{-1}(4x) + \sin^{-1}(3x) = -\frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}(4x) = -\frac{\pi}{2} - \sin^{-1}(3x)$

$\Rightarrow \quad 4x = -\sin\left(\frac{\pi}{2} + \sin^{-1} 3x\right)$

$= -\cos(\sin^{-1} 3x)$ **1**

$\Rightarrow \quad -4x = \sqrt{1-9x^2}$ **1/2**

$\Rightarrow \quad 16x^2 = 1 - 9x^2$

$\Rightarrow \quad 25x^2 = 1$

$\Rightarrow \quad x^2 = \frac{1}{25} \Rightarrow x = \pm\frac{1}{5}$

As $\sin^{-1} 4x + \sin^{-1} 3x < 0, x \neq \frac{1}{5}$ **1/2**

So, $x = -\frac{1}{5}$

OR

Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

Ans: $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$ **1**

$= \tan^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$ **1**

22. Express $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

Ans: $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$ **1/2**

$P = \frac{A+A^T}{2} = \frac{1}{2}\begin{bmatrix} 8 & -1 \\ -1 & -2 \end{bmatrix}$ **1/2**

$Q = \frac{A-A^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ **1/2**

Now, $A = P + Q$ **1/2**

$P + Q = \frac{1}{2}\begin{bmatrix} 8 & -6 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix} = A$

23. If $y^2 \cos\left(\frac{1}{x}\right) = a^2$, then find $\frac{dy}{dx}$.

Ans: $y^2 \cos\left(\frac{1}{x}\right) = a^2$

Then $2y\frac{dy}{dx}\cdot\cos\left(\frac{1}{x}\right) - y^2 \sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = 0$ **1**

$\Rightarrow \quad 2y\cdot\cos\left(\frac{1}{x}\right)\frac{dy}{dx} = -\frac{y^2}{x^2}\sin\left(\frac{1}{x}\right)$

$\therefore \quad \frac{dy}{dx} = -\frac{y}{2x^2}\tan\left(\frac{1}{x}\right)$ **1**

24. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

Ans: $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$

$\Rightarrow \quad \left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}-\vec{b}\right|^2$

$\Rightarrow \quad 4\vec{a}\cdot\vec{b} = 0$ or $\vec{a}\cdot\vec{b} = 0$ or $\vec{a} \perp \vec{b}$ **1**

Let $\vec{a} \perp \vec{b}$

Then $\vec{a}\cdot\vec{b} = 0$

Thus, $\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2$ and $\left|\vec{a}-\vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2$

$\Rightarrow \quad \left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$ **1**

OR

Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, 3\hat{i}+7\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+2\hat{k}$ form the sides of a right-angled triangle.

Ans: Let $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{b} = 3\hat{i}+7\hat{j}+\hat{k}$ and $\vec{c} = 5\hat{i}+6\hat{j}+2\hat{k}$

Since $\vec{c} = \vec{a}+\vec{b}$, three vectors form a triangle. **1**

Also, $\vec{a}\cdot\vec{b} = 0$.

So, triangle is a right angled triangle. **1**

25. Find the coordinates of the point where the line through (-1, 1, –8) and (5, –2, 10) crosses the ZX-plane.

Ans: Let the line segment AB is cut by ZX-plane in the ratio $1 : \lambda$. So, y-coordinate is zero. **1**

i.e., $\dfrac{-2+\lambda}{1+\lambda} = 0$ i.e. $\lambda = 2$

$\therefore$ The point of intersection is (1, 0, –2) **1**

26. If A and B are two events such that P(A) = 0.4, P(B) = 0.3 and $P(A \cup B) = 0.6$, then find $P(B' \cap A)$.

Ans: $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.1$ **1**

$P(B' \cap A) = P(A) - P(A \cap B) = 0.3$ **1**

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. Show that the function f: $(-\infty, 0) \to (-1, 0)$ defined by

$f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$ is one-one and onto.

Ans: Let $x_1, x_2 \in (-\infty, 0)$ such that $f(x_1) = f(x_2)$

i.e., $\frac{x_1}{1+|x_1|} = \frac{x_2}{1+|x_2|}$

$\Rightarrow \quad \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$ **1**

$\Rightarrow \quad x_1 - x_1x_2 = x_2 - x_1x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one. **1**

Let y ∈ (–1, 0) such that $y = \frac{x}{1+|x|}$

$\Rightarrow \quad y = \frac{x}{1-x}$

$\Rightarrow \quad x = \frac{y}{1+y}$ **1**

For each y ∈ (–1, 0), there exists x ∈ (–∞, 0),

such that $f(x) = f\left(\frac{y}{1+y}\right) = \frac{\frac{y}{1+y}}{1+\left|\frac{y}{1+y}\right|}$

$= \frac{\frac{y}{1+y}}{1-\frac{y}{1+y}} = y$

Hence f is onto. **1**

OR

Show that the relation R in the set A = {1, 2, 3, 4, 5, 6} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation.

Ans: **Reflexive:** $|a-a| = 0$, which is divisible by 2 for all a ∈ A.

$\therefore$ (a, a) ∈ R ⇒ R is reflexive. **1**

Symmetric: Let (a, b) ∈ R i.e., $|a-b| = 2\lambda$, $\lambda \in \omega$

then $|b-a| = |-(a-b)| = |a-b| = 2\lambda$

$\Rightarrow$ (b, a) $\in$ R $\Rightarrow$ R is symmetric. **1**

Transitive : Let (a, b), (b, c) $\in$ R i.e., $|a-b| = 2\lambda$, $|b-c| = 2\mu$

$$a-c = (a-b)+(b-c) = \pm 2\lambda \pm 2\mu = \pm 2(\lambda+\mu)$$

$|a-c| = 2|\lambda+\mu|$, which is divisible by 2

$\Rightarrow$ (a, c) $\in$ R $\Rightarrow$ R is transitive. **1**

Hence R is an equivalence relation. **1**

28. If $y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

Ans: Let $u = x^3(\cos x)^x$ and $v = \sin^{-1}\sqrt{x}$ so that y = u + v

$\log u = 3\log x + x\log(\cos x)$ **1/2**

$\Rightarrow \quad \frac{1}{u}\frac{du}{dx} = \frac{3}{x} - x\tan x + \log\cos x$ **1**

$\Rightarrow \quad \frac{du}{dx} = x^3(\cos x)^x\left[\frac{3}{x} - x\tan x + \log\cos x\right]$ .... (i) **1/2**

and $v = \sin^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$ ... (ii) **1**

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$\Rightarrow \quad \frac{dy}{dx} = x^3(\cos x)^x\left[\frac{3}{x} - x\tan x + \log\cos x\right] + \frac{1}{2\sqrt{x-x^2}}$ **1**

29. Evaluate: $\int_{-1}^{5}(|x|+|x+1|+|x-5|)\,dx$

Ans: If $x \in [-1,0] \Rightarrow f(x) = -x+x+1-x+5 = 6-x$ **1**

If $x \in [0,5] \Rightarrow f(x) = x+x+1-x+5 = x+6$ **1**

$\therefore \int_{-1}^{5}\left(|x|+|x+1|+|x-5|\right)dx = \int_{-1}^{0}(6-x)dx + \int_{0}^{5}(x+6)dx$ **1**

$$= \left[\frac{(6-x)^2}{-2}\right]_{-1}^{0} + \left[\frac{(x+6)^2}{2}\right]_{0}^{5}$$

$= \frac{13}{2} + \frac{85}{2} = 49$ **1**

30. Find the general solution of the differential equation $x^2y\,dx - \left(x^3 + y^3\right)dy = 0$

Ans: $\frac{dx}{dy} = \frac{x^3 + y^3}{x^2y}$

Put $x = vy \Rightarrow \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$ **1**

$\therefore \quad v + y\frac{dv}{dy} = \frac{y^3(v^3+1)}{y^3v^2}$ **1**

$\Rightarrow \quad y\frac{dv}{dy} = \frac{1}{v^2}$

$\Rightarrow \quad v^2 dv = \frac{dy}{y}$ **1**

Integrating both sides, we get

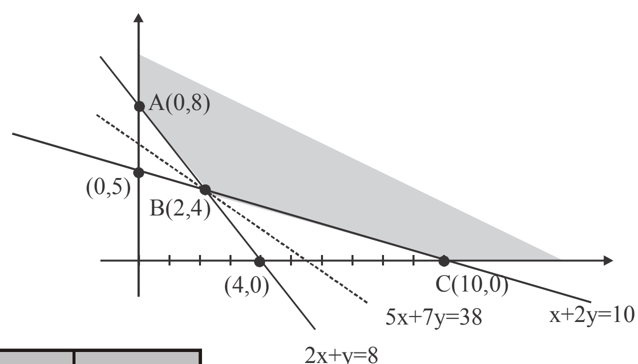
$\frac{v^3}{3} = \log y + c \Rightarrow \frac{x^3}{3y^3} = \log y + c$ **1**

$\Rightarrow x^3 = 3y^3 \log y + 3cy^3$

31. Solve the following LPP graphically:
Minimise z = 5x + 7y
subject to the constraints

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x, y \geq 0$$

Ans:



1+1

Corner Points	Z	
A (0, 8)	56	
B (2, 4)	38	← Smallest value
C (10, 0)	50	

$1\frac{1}{2}$

To verify whether the smallest value of z = 38 is the minimum value we draw open half plane.

5x + 7y < 38. Since there is no common point with the possible feasible region except (2, 4).

Hence minimum value of z = 38 at x = 2 and y = 4. **1/2**

32. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin?

Ans: Let E_1 be the event that unbiased coin is tossed.
E_2 be the event that biased coin is tossed.
A be the event that coin tossed shows tail } **1**

$$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{2}, P(A|E_1)=\frac{1}{2}, P(A|E_2)=\frac{2}{5}$$ **1**

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$ **1**

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{2}{5}}=\frac{5}{9}$$ **1**

OR

The probability distribution of a random variable X, where k is a constant is given below:

$$P(X=x)=\begin{cases}0\cdot1, & \text{if} \quad x=0\\ kx^2, & \text{if} \quad x=1\\ kx, & \text{if} \quad x=2 \text{ or } 3\\ 0, & \text{otherwise}\end{cases}$$

Determine

(a) the value of k

(b) $P(x \le 2)$

(c) Mean of the variable X.

Ans:

x_i	P_i
0	0.1
1	k
2	2k
3	3k

(i) $\sum P_i = 1$

$\Rightarrow 0\cdot1 + 6k = 1$

$\Rightarrow \quad k=\frac{3}{20}$ **1**

(ii) $P(x\le 2)=0.1+3k$

$=\frac{1}{10}+\frac{9}{20}=\frac{11}{20}$ **$1\frac{1}{2}$**

(iii) Mean = $\sum P_i x_i = 14k = \frac{21}{10}$ **$1\frac{1}{2}$**

SECTION-D

Question numbers 33 to 36 carry 6 marks each.

33. Solve the following system of equations by matrix method:

$$x - y + 2z = 7$$
$$2x - y + 3z = 12$$
$$3x + 2y - z = 5$$

Ans: Writing given equations in matrix form

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$$

Which is of the form AX = B **1**

Here $|A| = -2 \neq 0$ **1**

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$$ **2**

$$\therefore \quad X = A^{-1}B = \frac{1}{-2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$ **1**

$\Rightarrow \quad x = 2, y = 1, z = 3$ **1**

OR

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$

Ans: Using elementary row transformation,

$$A = IA \Rightarrow \begin{bmatrix} 2 & 1 & -3 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ **1**

Operating $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

[4 marks for correct operations]

$R_2 \to R_2 + R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 5 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ -3 & -3 & 1 \end{bmatrix} \cdot A$$

$R_2 \to - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -5 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -2 & 0 \\ -3 & -3 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + R_3$, $R_2 \to R_2 - 5R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 1 \\ 14 & 13 & -5 \\ -3 & -3 & 1 \end{bmatrix} \cdot A$$

$R_3 \to - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 1 \\ 14 & 13 & -5 \\ 3 & 3 & -1 \end{bmatrix} \cdot A$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} -2 & -2 & 1 \\ 14 & 13 & -5 \\ 3 & 3 & -1 \end{bmatrix}$$ **1**

34. Find the points on the curve $9y^2 = x^2$, where the normal to the curve makes equal intercepts with both the axes. Also find the equation of the normals.

Ans: Equation of given curve, $9y^2 = x^3$ ... (i)

$$\Rightarrow \quad 18y\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{x^2}{6y}$$ **1/2**

Slope of normal $= \dfrac{-6y}{x^2}$ **1/2**

$$-\frac{6y}{x^2} = \pm 1 \quad \text{(given)}$$

$\Rightarrow \quad y = \pm\dfrac{x^2}{6}$ ... (ii) **1**

From (i) & (ii), we get

$9\cdot\dfrac{x^4}{36} = x^3 \Rightarrow x^3(x-4) = 0 \Rightarrow x = 0, 4$ (x = 0 is rejected)

$x = 4, y^2 = \dfrac{64}{9} \Rightarrow y = \pm\dfrac{8}{3}$ **1**

Point of contacts are $\left(4, \dfrac{8}{3}\right), \left(4, \dfrac{-8}{3}\right)$ **$1\frac{1}{2}$**

Equation of normal at $\left(4, \dfrac{8}{3}\right)$ is $y - \dfrac{8}{3} = -(x-4)$

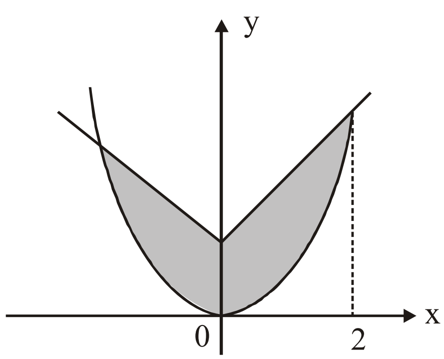
$\Rightarrow 3x + 3y - 20 = 0$ **1**

and equation of normal at $\left(4, -\dfrac{8}{3}\right)$ is $y + \dfrac{8}{3} = -(x-4)$

$\Rightarrow 3x + 3y = 20$ **1/2**

35. Find the area of the following region using integration: $\{(x, y) : y \le |x| + 2, y \ge x^2\}$.

Ans: **[Correct figure and shade (2)]**



$y = x^2$

$y = |x| + 2 = x + 2, \text{ if } x \ge 0$

$= -x + 2, \text{ if } x < 0$

Solving, $y = x^2$ and $y = x + 2$

$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2, -1$ (x = –1 rejected) **1**

Required area $= 2\left[\displaystyle\int_0^2 (x+2)dx - \int_0^2 x^2 dx\right]$ **1**

$= 2\left[\dfrac{(x+2)^2}{2} - \dfrac{x^3}{3}\right]_0^2$ **1**

$= 2\left[6 - \dfrac{8}{3}\right] = \dfrac{20}{3}$ sq. units **1**

OR

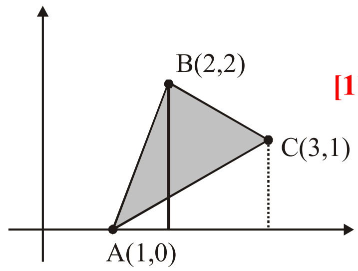
Using integration, find the area of a triangle whose vertices are (1,0), (2, 2) and (3,1).

Ans:

Equations of AB; $y = 2x - 2$

BC; $y = 4 - x$

AC; $y = \frac{1}{2}x - \frac{1}{2}$



[1½ marks for correct equations]

[1 mark for figure and shade]

Required area $= 2\int_1^2 (x-1)dx + \int_2^3 (4-x)dx - \frac{1}{2}\int_1^3 (x-1)dx$ **$1\frac{1}{2}$**

$= 2\left[\frac{(x-1)^2}{2}\right]_1^2 - \left[\frac{(4-x)^2}{2}\right]_2^3 - \frac{1}{2}\left[\frac{(x-1)^2}{2}\right]_1^3$ **1**

$= 1 + \frac{3}{2} - 1 = \frac{3}{2}$ sq. units **1**

36. Show that the lines

$\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-4}{2}$ intersect.

Also, find the coordinates of the point of intersection. Find the equation of the plane containing the two lines.

Ans: $\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-3}{1} = \lambda$ (say)

and $\frac{x-2}{1} = \frac{y-3}{3} = \frac{z-4}{2} = \mu$ (say)

Arbitrary points on the lines are

$(\lambda + 2, 3\lambda + 2, \lambda + 3)$ and $(\mu + 2, 4\mu + 3, 2\mu + 4)$

$\Rightarrow$ $\lambda + 2 = \mu + 2$, and $\lambda + 3 = 2\mu + 4$

$\Rightarrow$ $\lambda = \mu$, solving we get $\lambda = -1$, $\mu = -1$ **1**

$\lambda = -1$, $\mu = -1$ satisfying y-coordinates $3\lambda + 2 = 4\mu + 3$ **1**

$\therefore$ Point of intersection is $(1, -1, 2)$ **1**

Equation of plane passing through two given lines are **1**

$$\begin{vmatrix} x-2 & y-2 & z-3 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix} = 0$$ **1**

$\Rightarrow \quad 2x - y + z - 5 = 0$ **1**