QUESTION PAPER CODE 65/3/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} - \frac{2}{x} \cdot y = 2x \Rightarrow \text{I.F.} = e^{-2\log x} = \frac{1}{x^2}$ $\qquad \frac{1}{2} + \frac{1}{2}$

2. $y^2 + 2xy\frac{dy}{dx} - 2x = 0 \Rightarrow \frac{dy}{dx} = \frac{2x - y^2}{2xy}$ $\qquad \frac{1}{2} + \frac{1}{2}$

3. $|-2A| = (-2)^3 \cdot |A|$ $\qquad \frac{1}{2}$

$= -8 \times 4 = -32$ $\qquad \frac{1}{2}$

4. $\sqrt{(-18)^2 + (12)^2 + (-4)^2} = 22$ $\qquad \frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\qquad \frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\qquad \frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\qquad \frac{1}{2}$

SECTION B

5. Let a = 2, b = 3 $\Rightarrow 2*3 = \frac{2}{4} = \frac{1}{2}$, $3*2 = \frac{3}{3} = 1 \Rightarrow 2*3 \neq 3*2.$ $\qquad 1$

$$(2*3)*4 = \frac{1}{2}*4 = \frac{\frac{1}{2}}{4+1} = \frac{1}{10}, 2*(3*4) = 2*\frac{3}{5} = \frac{2}{8/5} = \frac{5}{4}$$

$\Rightarrow (2*3)*4 \neq 2*(3*4)$ $\qquad 1$

6. $A^2 = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = A$ $\qquad 1$

$\Rightarrow A^3 = A^2 \cdot A = A \cdot A = A^2 = A$ $\qquad 1$

7. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\qquad \frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = -m \quad \text{...(ii)}$$ $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

8. $\int\frac{\sin x-\cos x}{\sqrt{1+\sin 2x}}dx = \int\frac{\sin x-\cos x}{\sin x+\cos x}dx$ 1

$= -\log|\sin x+\cos x|+c$ 1

9. $\int\frac{\sin(x-a)}{\sin(x+a)}dx = \int\frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int\left[\frac{\sin(x+a)\cdot\cos 2a}{\sin(x+a)}-\frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a-\sin 2a\cdot\log|\sin(x+a)|+c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int(\log x)^2\cdot 1dx = (\log x)^2\cdot x-\int 2\cdot\log x\cdot\frac{1}{x}\cdot x\,dx$ 1

$= x\cdot(\log x)^2-\left\{\log x\cdot 2x-\int\frac{1}{x}\cdot 2xdx\right\}$ $\frac{1}{2}$

$= x(\log x)^2-2x\log x+2x+c$ $\frac{1}{2}$

10. A = {(S, F, M), (S, M, F), (M, F, S), (F, M, S)}
B = {(S, F, M), (M, F, S)} 1

Total number of possible arrangements = 6

$P(B|A) = \frac{P(B\cap A)}{P(A)}$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$ 1

11. Given $2\,P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}$ and $P(X = x_4) = \frac{k}{5}$ $\frac{1}{2}$

$\therefore\ k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$ 1

12. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a} \times \vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j} + \hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a} \cdot \vec{b} \times \vec{c} = 0$ $\frac{1}{2}$

$$\left.\begin{aligned} \vec{a} \cdot \vec{b} \times \vec{c} &= \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3) \\ &= 3 - 12 + 9 = 0 \end{aligned}\right\}$$ $1 + \frac{1}{2}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

SECTION C

13. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let (a, b) $\in$ R for a, b $\in$ Z, then a – b is divisible by 2

$\Rightarrow$ (b – a) is also divisible by 2

$\therefore$ (b, a) $\in$ R $\Rightarrow$ R is symmetric ...(ii) 1

For a, b, c, $\in$ Z, Let (a, b) $\in$ R and (b, c) $\in$ R

$\therefore$ a – b = 2p, p $\in$ Z, and b – c = 2q, q $\in$ Z,

adding, a – c = 2 (p + q) $\Rightarrow$ (a – c) is divisible by 2

$\Rightarrow$ (a, c) $\in$ R, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$$\text{fof}(x) = f\left(\frac{4x+3}{6x-4}\right)$$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow \text{fof}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x $\Rightarrow$ fof = I $\Rightarrow f^{-1} = f$ 1

14. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \dfrac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log \sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1-y(x+y)\cot x}{(x+y)\log \sin x - 1}$$ $\frac{1}{2}$

15. $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{2} - \sin^{-1}\frac{4}{x} = \cos^{-1}\frac{4}{x}$ 1

$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\sqrt{1-\frac{16}{x^2}}\right) \Rightarrow \left(\frac{3}{x}\right)^2 = \frac{x^2-16}{x^2}$ $1\frac{1}{2}$

$\Rightarrow x^2 = 25 \Rightarrow x = \pm 5,\ x = -5$ (rejected) $\therefore\ x = 5$ $\frac{1}{2}+1$

16. $\text{LHS} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix}$ $\left\{\begin{array}{l}\text{Applying} \\ R_1 \to aR_1 \\ R_2 \to bR_2 \\ R_3 \to cR_3\end{array}\right.$ 1

$= \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$ $\{R_1 \to R_1 + R_2 + R_3$ $\frac{1}{2}+1$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = 1+a^2+b^2+c^2. \;=\; \text{RHS} \quad \begin{cases} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{cases}$$ $1\frac{1}{2}$

17. $y = (\cot^{-1} x)^2 \;\Rightarrow\; \dfrac{dy}{dx} = 2\cot^{-1} x \cdot \left(\dfrac{-1}{1+x^2}\right)$ 1

$\Rightarrow (1+x^2)\dfrac{dy}{dx} = -2\cot^{-1} x = -2\sqrt{y}$ $\frac{1}{2}$

squaring both sides, we get

$(1+x^2)^2 \cdot \left(\dfrac{dy}{dx}\right)^2 = 4y$ $\frac{1}{2}$

differentiating, w.r.t. x,

$2(1+x^2)2x \cdot \left(\dfrac{dy}{dx}\right)^2 + 2(1+x^2)^2 \cdot \dfrac{dy}{dx} \cdot \dfrac{d^2y}{dx^2} = 4 \cdot \dfrac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow 2x(1+x^2)\dfrac{dy}{dx} + (1+x^2)^2 \dfrac{d^2y}{dx^2} = 2.$ $\frac{1}{2}$

18. $I = \displaystyle\int \dfrac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)}\,dx$

Put $\sin^2 x = t \;\Rightarrow\; \sin 2x \, dx = dt$ $\frac{1}{2}$

$\therefore\; I = \displaystyle\int \dfrac{dt}{(t+1)(t+3)} = \int\left(\dfrac{1/2}{t+1} + \dfrac{-1/2}{t+3}\right)dt$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log|t+1| - \dfrac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log(\sin^2 x+1) - \dfrac{1}{2}\log(\sin^2 + 3) + c.$ $\frac{1}{2}$

19. RHS $= \displaystyle\int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \displaystyle\int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \text{LHS}$ $\frac{1}{2}$

Let I $= \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx$ ...(i) $\frac{1}{2}$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\, dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\pi/6}^{\pi/3} 1\cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

20. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2} dv - \frac{1}{2}\int \frac{2v}{1+v^2} dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\, dx = \cot x.dx.$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\cot x}{1+x^2}$$ 1

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$$ 1

$\therefore$ Solution is, $y \cdot (1+x^2) = \int \cot x \, dx = \log|\sin x| + c$ 1+1

or $y = \frac{1}{1+x^2} \cdot \log|\sin x| + \frac{c}{1+x^2}$

21. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$$\Rightarrow (5\lambda+2)\cdot 1 + (-5)(2\lambda) + 1(3) = 0$$ $\frac{1}{2}$

$$-5\lambda + 5 = 0 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7} = \frac{y-2}{-5} - \frac{z-1}{1}; \frac{x}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $= \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{\left|\left(5\hat{i} + \frac{5}{2}\hat{j}\right)\cdot(-17\hat{i} - 20\hat{j} + 19\hat{k})\right|}{|\vec{b}_1 \times \vec{b}_2|}$ 1

$$= \frac{135}{|\vec{b}_1 \times \vec{b}_2|} \neq 0$$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} \therefore \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b} \perp \vec{c} \Rightarrow \vec{b}\cdot\vec{c} = 0$ ...(ii) $\frac{1}{2}$

$$(|3\vec{a} - 2\vec{b} + 2\vec{c}|)^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 - 12\,\vec{a}\cdot\vec{b} - 8\,\vec{b}\cdot\vec{c} + 12\,\vec{a}\cdot\vec{c}$$ 1

$= 9(1)^2 + 4(2)^2 + 4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61}$ 1

23. Curve $y = \frac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$\frac{dy}{dx} = \frac{(x^2 - 5x + 6) \cdot 1 - (x-7)(2x-5)}{(x^2 - 5x + 6)^2}$ $\frac{1}{2}$

at (7, 0) $\frac{dy}{dx} = \frac{20}{(20)^2} = \frac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\frac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \frac{1}{20}(x - 7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x - 7)$

or $20x + y = 140$. $\frac{1}{2}$

SECTION D

24. $f(x) = \sin x + \frac{1}{2}\cos 2x \Rightarrow f'(x) = \cos x - \sin 2x$ 1

$f'(x) = 0 \Rightarrow \cos x - 2\sin x \cos x = 0$

$\Rightarrow \cos x(1 - 2\sin x) = 0$

$\Rightarrow x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}$ 1

$x = \frac{\pi}{6} \in \left(0, \frac{\pi}{2}\right)$ 1

$f''(x) = -\sin x - 2\cos 2x$ 1

$f''(\pi/6) < 0 \Rightarrow \quad x = \frac{\pi}{6}$ is a local maxima. 1

Local Max. Value = $f(\pi/6) = \frac{3}{4}$ 1

Local extreme values do exist at end points x = 0, x = $\frac{\pi}{2}$ but no marks are alotted here for that

25. $A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ 2

$A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ 3

or $A \cdot A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow A^2 = A^{-1}$ 1

OR

Given System of equation can be written as

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix}$ or $AX = B$ 1

$|A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$ 1

$\therefore X = A^{-1} \cdot B$

$(\text{adj. } A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ 2

$A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 2, y = 2, z = 3.$ $1\frac{1}{2}$

26. Let the events be

$$\left.\begin{array}{l} E_1 : \text{bag I is selected} \\ E_2 : \text{bag II is selected} \\ A : \text{getting a red ball} \end{array}\right\}$$ 1

$P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

$P(A/E_1) = \frac{3}{9} = \frac{1}{3}$; $P(A/E_2) = \frac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2} \cdot \frac{5}{5+n}}{\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{5}{5+n}}$$ 2

$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$\Rightarrow 16(x-2) + 24(y-5) + 32(z+3) = 0$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$\frac{x-3}{4} = \frac{y-1}{4} = \frac{z-5}{6}$ or $\frac{x-3}{2} = \frac{y-1}{2} = \frac{z-5}{3}$ ...(ii) 1

Any point on (ii) is $(2\lambda + 3, 2\lambda + 1, 3\lambda + 5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$, is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is || to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

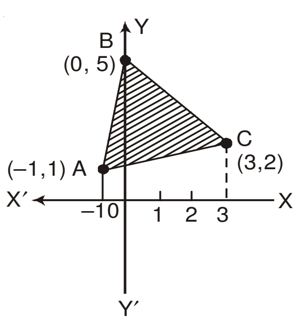
or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

28. Let the points be A (–1, 1), B (0, 5) and C (3, 2)

Correct Figure 1



Equation of AB : $y=4x+5$

BC : $y=5-x$

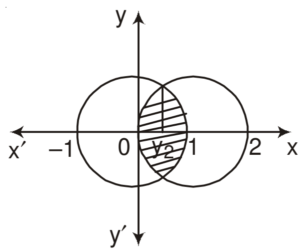
AC : $y=\frac{1}{4}(x+5)$ $1\frac{1}{2}$

Req. Area $=\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(5-x)dx-\int_{-1}^{3}\frac{1}{4}(x+5)dx$ $1\frac{1}{2}$

$$\therefore\ A = \left[\frac{(4x+5)^2}{8}\right]_{-1}^{0} + \left[\frac{(5-x)^2}{-2}\right]_{0}^{3} - \frac{1}{4}\left[\frac{(x+5)^2}{2}\right]_{-1}^{3}$$ 1

$$= 3 + \frac{21}{2} - 6 = \frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2 + y^2 = 1$$

and $x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$

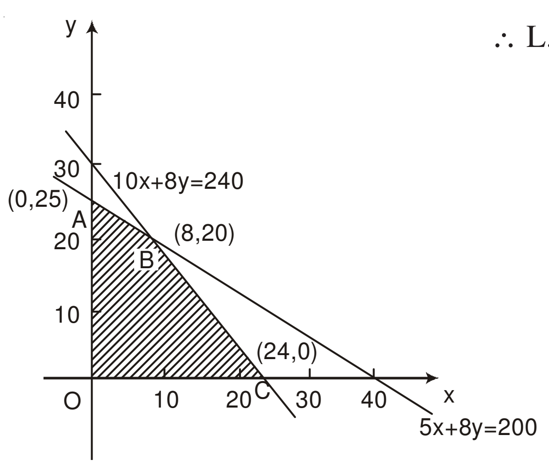
$$\Rightarrow x = \frac{1}{2}$$ 1

$$\therefore \text{ Required area } = 2\left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx\right]$$ 2

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$= 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] + 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that $5x + 8y \le 200$, $10x + 8y \le 240$, $x, y \ge 0$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.