QUESTION PAPER CODE 65/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION – A

Question Numbers 1 to 20 carry 1 mark each.

Question Numbers 1 to 10 are multiple choice type questions.
Select the correct option.

Q.No. **Marks**

1. If A is a 3 × 3 matrix and $|A| = -2$, then value of $|A\,(\text{adj } A)|$ is

(A) -2 **(B)** 2 **(C)** -8 **(D)** 8

Ans: (C) -8 1

2. The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

Ans: (A) 0 1

3. The principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

(A) $\frac{13\pi}{6}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{6}$

Ans: (D) $\frac{\pi}{6}$ 1

4. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4) and (0, 5). If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then

(A) $a = 2b$ **(B)** $2a = b$ **(C)** $a = b$ **(D)** $3a = b$

Ans: (A) $a = 2b$ 1

5. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B' \mid A)$ is equal to

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{4}$ **(D)** 1

Ans: (C) $\frac{3}{4}$ 1

6. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

(A) I **(B)** 0 **(C)** $I - A$ **(D)** $I + A$

Ans: (A) I 1

7. $\displaystyle\int_{-\frac{x}{2}}^{\frac{x}{2}} \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$, where $x \neq 0$ is equal to

(A) –2 (B) 0 (C) 1 (D) π

$\boxed{\textbf{Ans:}\ \text{(B)}\ 0}$ **1**

8. The image of the point (2, –1, 5) in the plane $\vec{r}\,.\,\hat{i} = 0$ is

(A) (–2, –1, 5) (B) (2, 1, –5) (C) (–2, 1, –5) (D) (2, 0, 0)

$\boxed{\textbf{Ans:}\ \text{(A)}\ (-2, -1, 5)}$ **1**

9. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(A) 0 (B) 1 (C) $\frac{-2}{3}$ (D) $\frac{-3}{2}$

$\boxed{\textbf{Ans:}\ \text{(C)}\ \frac{-2}{3}}$ **1**

10. The vector equation of the line passing through the point (–1 , 5, 4) and perpendicular to the plane z = 0 is

(A) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$ (B) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

(C) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$ (D) $\vec{r} = \lambda\hat{k}$

$\boxed{\textbf{Ans:}\ \text{(B)}\ \vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}}$ **1**

Fill in the blanks in questions numbers 11 to 15

11. The position vectors of two points A and B are $\overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectivley. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is __________.

$\boxed{\textbf{Ans:}\ 2\hat{i} - \hat{j} + \hat{k}}$ **1**

12. The equation of the normal to the curve $y^2 = 8x$ at the origin is __________.

$\boxed{\textbf{Ans:}\ y = 0}$ **1**

OR

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of __________ cm^2/s.

$\boxed{\textbf{Ans:}\ 12\pi}$ **1**

13. On applying elementary column operation $C_2 \to C_2 - 3C_1$ in the matriax equation $\begin{bmatrix} 4 & -2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, the RHS (Right Hand Side) of the equation becomes __________.

Ans: $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix}$ **1**

OR

A square matrix A is said to be symmetric if _______

Ans: $A = A'$ **1**

14. A relation R in a set A is called ________, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

Ans: Symmetric **1**

15. The greatest integer fucntion defined by $f(x) = [x]$, $0 < x < 2$ is not differentiable at x = ______.

Ans: 1 **1**

Question numbers 16 to 20 are very short answer type questions

16. If A is non-singular square matrix of order 3 and $A^2 = 2A$, then find the value of | A |.

Ans: $|A|^2 = 8|A|$ **1/2**

$\Rightarrow |A| = 8$ **1/2**

17. Two cards are drawn at random and one-by-one without replacement from a wll-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

Ans: $\frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26}{51}$ **1/2+1/2**

18. Evaluate $\int_1^3 |2x - 1|\,dx$.

Ans: $\int_1^3 |2x - 1|\,dx = \int_1^3 (2x - 1)\,dx = \left[\frac{1}{4}(2x - 1)^2\right]_1^3$ **1/2**

$= 6$ **1/2**

19. Find : $\int \frac{dx}{\sqrt{9-4x^2}}$

Ans: $\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{3^2-(2x)^2}}$ **1/2**

$= \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + C$ **1/2**

20. Find: $\int x^4 \log x \, dx$.

Ans: $\int x^4 \cdot \log x dx = \log x \cdot \frac{x^5}{5} - \int \frac{1}{x} \cdot \frac{x^5}{5} dx$ **1/2**

$= \frac{x^5 \cdot \log x}{5} - \frac{x^5}{25} + c$ **1/2**

OR

Find: $\int \frac{2x}{\sqrt[3]{x^2+1}} dx$.

Ans: Let, $x^2+1=t$

$\therefore \quad 2xdx = dt$ **1/2**

$$\int \frac{2x}{\sqrt[3]{x^2+1}} dx = \int \frac{1}{\sqrt[3]{t}} dt = \int t^{-1/3} dt = \frac{3}{2} t^{2/3} + c$$

$= \frac{3}{2}\left(x^2+1\right)^{2/3} + c$ **1/2**

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

Ans: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k}$ **1**

Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$ **1**

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}, -\vec{b}$ and $3\vec{c}$, where $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

Ans: Volume of the parallelopiped = $\begin{vmatrix} 2 & -2 & 4 \\ -3 & -4 & 5 \\ 6 & -3 & 9 \end{vmatrix}$ **1**

$= |-24| = 24$ **1**

22. Examine the applicability of Rolle's theorem for the function f(x) = sin 2x in [0, π]. Hence find the points where the tangent is parallel to x-axis.

Ans: As, sine function and polynomial function are everywhere continuous and differentiable.

∴ (i) f(x) = sin 2x is continuous on [0, π]

(ii) f(x) = sin 2x is differentiable on (0, π)

(iii) f(0) = 0 = f(π) **1**

∴ Rolle's Theorem is applicable for f(x) = sin2x

Solving, $f'(x) = 0$ or $2\cos 2x = 0 \Rightarrow \cos 2x = 0$

∴ $2x = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$ **1/2**

The points where the tangent is parallel to x-axis are: $\left(\frac{\pi}{4}, 1\right); \left(\frac{3\pi}{4}, -1\right)$ **1/2**

23. Find the values of x for which the fucntion f(x) = 2 + 3x – x^3 is decreasing.

Ans: f(x) is decreasing iff $f'(x) \le 0$ **1**

$\Leftrightarrow 3 - 3x^2 \le 0$ or $x^2 \ge 1$

$\Leftrightarrow x \le -1$ or $x \ge 1$ **1**

24. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

Ans: Probability of green signal on crossing $X = \frac{30}{100} = \frac{3}{10}$ **1**

Probability of not a green signal on crossing $X = 1 - \frac{3}{10} = \frac{7}{10}$

Probability of a green signal on X on two concecutative days out of three

$= \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{63}{500}$ **1**

25. Prove that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\cos^{-1}x, \frac{1}{\sqrt{2}} \le x \le 1$

Ans: Put $x = \cos\theta \Leftrightarrow \theta = \cos^{-1}x$ **1/2**

L.H.S. $= \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= \sin^{-1}(2\cos\theta\ \sin\theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\cos^{-1}x = \text{R.H.S.}$ $1\frac{1}{2}$

OR

Consider a bijective function $f: R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.

Ans: Let $y = f(x) = 16x^2 + 24x + 7 = (4x + 3)^2 - 2$ **1**

$\Rightarrow\ f^{-1}(y) = x = \frac{\sqrt{y+2}-3}{4}$ **1**

26. Find the value of k so that the lines x = –y = kz and x – 2 = 2y + 1 = –z + 1 are perpendicular to each other.

Ans: The lines, $\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$ and $\frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$ **1**

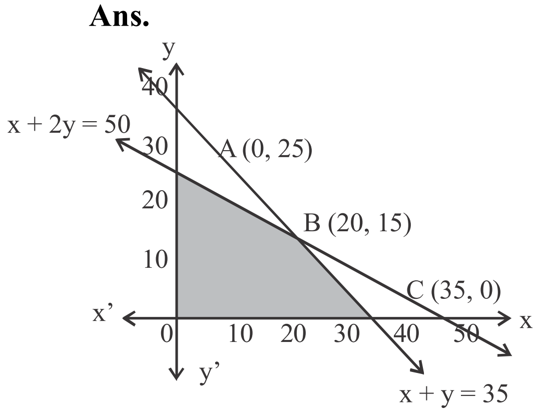
are perpendicular $\therefore\ 1 - \frac{1}{2} - \frac{1}{k} = 0 \ \Rightarrow k = 2$ **1**

SECTION-C

Question numbers 27 to 32 carry 4 marks each.

27. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

Ans.



Let No. of chairs = x, No. of tables = y

Then L.P. P. is:

Maximize (Profit) : Z = 150x + 250y **1**

Subject to: $x + y \le 35$
$1000x + 2000y \le 50000 \Rightarrow x + 2y \le 50$
$x, y \ge 0$ **1**

Correct graph $1\frac{1}{2}$

Corner:	Value of Z
A(0, 25)	₹ 6250
B(20, 15)	₹ 6750 (Max)
C(35, 0)	₹ 5250

$\therefore$ Max (Z) = ₹ 6750

Number of chairs = 20, Tables = 15 — 1/2

28. If $x = a\sec^3\theta$, $y = a\tan^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Ans. $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta} = \sin\theta$ — 2

$\frac{d^2y}{dx^2} = \cos\theta\cdot\frac{d\theta}{dx} = \frac{1}{3a\sec^4\theta\cdot\tan\theta}$ — $1\frac{1}{2}$

$\left.\frac{d^2y}{dx^2}\right|_{\theta=\frac{\pi}{4}} = \frac{1}{12a}$ — 1/2

29. Find: $\int\frac{2x+1}{\sqrt{3+2x-x^2}}dx$.

Ans. $\int\frac{2x+1}{\sqrt{3+2x-x^2}}dx = -\int\frac{2-2x}{\sqrt{3+2x-x^2}}dx + 3\int\frac{1}{\sqrt{2^2-(x-1)^2}}dx$ — 2

$= -2\sqrt{3+2x-x^2} + 3\sin^{-1}\left(\frac{x-1}{2}\right) + c$ — 2

30. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

Ans. E_1 = Event that the ball transfered from Bag I is Black

E_2 = Event that the ball transfered from Bag I is Red

A = Event that the ball drawn from Bag II is Black — 1/2

$P(E_1) = \frac{5}{8}; P(E_2) = \frac{3}{8}; P\left(\frac{A}{E_1}\right) = \frac{4}{8} = \frac{1}{2}; P\left(\frac{A}{E_2}\right) = \frac{3}{8}$ — 2

Required Probability:

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)} = \frac{\frac{5}{8}\cdot\frac{1}{2}}{\frac{5}{8}\cdot\frac{1}{2}+\frac{3}{8}\cdot\frac{3}{8}} = \frac{20}{29}$$ $1\frac{1}{2}$

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

Ans. Let X = No. of white balls = 0, 1, 2

X :	0	1	2	1/2
P(X) :	$\frac{8}{10}\times\frac{7}{9}\times\frac{6}{8}=\frac{7}{15}$	$3\times\frac{8}{10}\times\frac{7}{9}\times\frac{2}{8}=\frac{7}{15}$	$3\times\frac{2}{10}\times\frac{1}{9}\times\frac{8}{8}=\frac{1}{15}$	$1\frac{1}{2}$
$X\cdot P(X)$:	0	$\frac{7}{15}$	$\frac{2}{15}$	1/2
$X^2P(X)$:	0	$\frac{7}{15}$	$\frac{4}{15}$	

Mean $= \sum XP(X) = \frac{9}{15} = \frac{3}{5}$ 1/2

Variance $= \sum X^2P(x) - \left[\sum X\,P(X)^2\right] = \frac{11}{15} - \left[\frac{3}{5}\right]^2 = \frac{28}{75}$ 1

31. Find the general solution of the differential equation $ye^y dx = (y^3 + 2xe^y)dy$.

Ans. $y\cdot e^y dx = \left(y^3 + 2xe^y\right)dy \Rightarrow y\cdot e^y \frac{dy}{dx} = y^3 + 2xe^y$

$\therefore \quad \frac{dx}{dy} - \frac{2}{y}x = y^2\cdot e^{-y}$ 1

I.F. (Integrating factor) $= e^{-2\int\frac{1}{y}dy} = e^{-2\log y} = e^{\log\frac{1}{y^2}} = \frac{1}{y^2}$ 1

$\therefore$ Solution is

$$x\cdot\frac{1}{y^2} = \int y^2\cdot e^{-y}\cdot\frac{1}{y^2}dy + c = \int e^{-y}dy + c$$ 1

$\Rightarrow \frac{x}{y^2} = -e^{-y} + c$ or $x = -y^2e^{-y} + cy^2$ 1

OR

Find the particular solution of the differential equation

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right), \text{ given that } y = \frac{\pi}{4} \text{ at } x = 1.$$

Ans. The differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}, \text{ let } y = vx \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$ **1**

$$\Rightarrow v + x\frac{dv}{dx} = v - \tan v \Rightarrow \cot v \, dv = -\frac{1}{x}dx$$

Integrate both sides

$$\log\sin v = -\log|x| + \log c \Rightarrow \log\sin\frac{y}{x} = \log\frac{c}{x}$$ **2**

$$\Rightarrow x\cdot\sin\frac{y}{x} = c, \text{ Put } y = \frac{\pi}{4} \text{ and } x = 1$$

$$\Rightarrow \sin\frac{\pi}{4} = c \text{ or } c = \frac{1}{\sqrt{2}}$$ **1/2**

$$\therefore \text{ Particular solution is } x\cdot\sin\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}}$$ **1/2**

32. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d ∈ N. Show that R is an equivalence relation.

Ans: Reflexive: For any (a, b) ∈ N × N

$a \cdot b = b \cdot a$

∴ (a, b) R (a, b) thus R is reflexive **1**

Symmetric: For (a, b), (c, d) ∈ N × N

(a, b) R (c, d) $\Rightarrow a \cdot d = b \cdot c$

$\Rightarrow c \cdot b = d \cdot a$

$\Rightarrow$ (c, d) R (a, b) ∴ R is symmetric **$1\frac{1}{2}$**

Transitive : For any (a, b), (c, d), (e, f), ∈ N × N

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow a \cdot d = b \cdot c$ and $c \cdot f = d \cdot e$

$\Rightarrow a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$

∴ (a, b) R (e, f), ∴ R is transitive **$1\frac{1}{2}$**

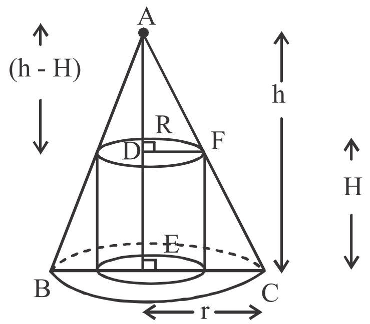
∴ R is an equivalance Relation

SECTION-D

Question numbers 33 to 36 carry 6 marks.

33. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

Ans.



Let H = Height of cylinder

R = Radius of cylinder

Volume of cone = $\frac{\pi}{3}r^2h$ **1/2**

V = Volume of cylinder = $\pi R^2 H$ **1/2**

$\Delta ADF \sim \Delta AEC \Rightarrow \frac{h-H}{h} = \frac{R}{r} \Rightarrow R = \frac{r}{R}(h-H)$ **1**

$\therefore V = \pi \cdot H \cdot \frac{r^2}{h^2}(h-H)^2 = \frac{\pi r^2}{h^2}\left(H^3 - 2hH^2 + Hh^2\right)$ **1**

$V'(H) = \frac{\pi r^2}{h^2}\left(3H^2 - 4hH + h^2\right),\ V'(h) = 0 \Rightarrow H = \frac{h}{3}$ **1+1**

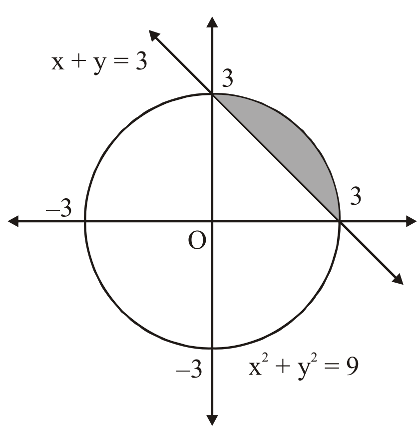
$V''(H) = \frac{\pi r^2}{h^2}(6H-4h),\ V''\left(H = \frac{h}{3}\right) = \frac{\pi r^2}{h^2}(-2h) < 0$ **1/2**

$\therefore$ V is max iff $H = \frac{h}{3}$ and $R = \frac{2r}{3}$

$$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{3\pi R^2 H}{\pi r^2 h} = 3 \cdot \frac{4r^2}{9} \cdot \frac{h}{3} \cdot \frac{1}{r^2 h} = \frac{4}{9}$$ **1/2**

34. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \le 9, x + y \ge 3\}$

Ans. Correct graph. **2**



Required area

$$= \int_0^3 \sqrt{9-x^2}\,dx - \int_0^3 (3-x)\,dx$$ **2**

$$= \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 + \left[\frac{1}{2}(3-x)^2\right]_0^3$$ **$1\frac{1}{2}$**

$$= \frac{9\pi}{4} - \frac{9}{2} \text{ or } \frac{9}{4}(\pi - 2)$$ **1/2**

35. Find the equation of the plane that contains the point A(2,1,–1) and is perpendicular to the line of intersection of the planes $2x+y-z=3$ and $x+2y+z=2$. Also find the angle between the plane thus obtained and the y-axis.

Ans. Let equation of the required plane be:

$a(x-2)+b(y-1)+c(z+1)=0$ **$1\frac{1}{2}$**

Also : $2a+b-c=0$

$a+2b+c=0$

Solving: $\frac{a}{3}=\frac{b}{-3}=\frac{c}{3}=k \Rightarrow a=3k,\ b=-3k,\ c=3k$ **$1\frac{1}{2}$**

$\therefore$ Equation of plane is : $3k(x-2)-3k(y-1)+3k(z+1)=0$

$\Rightarrow x-y+z=0$ **$1\frac{1}{2}$**

Let angle between y-axis and plane = θ

then, $\sin\theta=\left|\frac{0-1+0}{\sqrt{1+1+1}}\right|=\left|\frac{-1}{\sqrt{3}}\right| \Rightarrow \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **$1\frac{1}{2}$**

OR

Find the distance of the point P(–2, –4, 7) from the point of intersection Q of the line $\vec{r}=(3\hat{i}-2\hat{j}+6\hat{k})+\lambda(2\hat{i}-\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=6$. Also write the vector equation of the line PQ.

Ans. General point on line is: $\vec{r}=(3+2\lambda)\hat{i}+(-2-\lambda)\hat{j}+(6+2\lambda)\hat{k}$ **1**

For the point of intersection:

$\left[(3+2\lambda)\hat{i}+(-2-\lambda)\hat{j}+(6+2\lambda)\hat{k}\right]\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=6$ **1**

$\Rightarrow 3+2\lambda+2+\lambda+6+2\lambda=6 \Rightarrow \lambda=-1$ **1**

$\therefore Q(\hat{i}-\hat{j}+4\hat{k})=Q(1,-1,4)$ **1**

$PQ=3\sqrt{3}$, equation of the line PQ : $\vec{r}=-2\hat{i}-4\hat{j}+7\hat{k}+\mu\left(3\hat{i}+3\hat{j}-3\hat{k}\right)$ **1+1**

36. If $A=\begin{bmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations:

$x+2y-3z=6$

$3x+2y-2z=3$

$2x-y+z=2$

Ans. $|A| = 7$; $\text{adj}(A) = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$; $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$ $\quad 1 + 1\frac{1}{2} + \frac{1}{2}$

The system of equations in Matrix form can be written as :

$A \cdot X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$ $\quad 1$

$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$ $\quad 1$

$\therefore \; x = 1, y = -5, z = -5$ $\quad 1$

OR

Using properties of determinants, prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

Ans. $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$= \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ca \\ a^2+b^2 & c^2 & ab \end{vmatrix}$ $\quad (C_1 \to C_1 - 2C_{3)}$ $\quad 1$

$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$ $\quad (C_1 \to C_1 + C_2)$ $\quad 1$

$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$ $\quad (R_2 \to R_2 - R_1, R_3 \to R_3 - R_1)$ $\quad 2$

$$= (b-a)(c-a)\begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix}$$ **1**

Expand along C_1

$$= \left(a^2+b^2+c^2\right)(b-a)\,(c-a)\left(-b^2-ab+c^2+ac\right)$$

$$= (a-b)\,(b-c)\,(c-a)\,(a+b+c)\left(a^2+b^2+c^2\right)$$ **1**