QUESTION PAPER CODE 65/4/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|2\text{ adj }A| = 2^3\ |A|^{3-1} = 8 \times 81 = 648$ $\frac{1}{2}+\frac{1}{2}$

2. $\cos\theta = \frac{3+12-4}{3\times 7} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\right)$ $\frac{1}{2}+\frac{1}{2}$

OR

$\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1$, length of x − intercept $=\frac{5}{2}$ 1

3. $\frac{dy}{dx} = +\frac{\operatorname{cosec}(\cot\sqrt{x})\cot(\cot\sqrt{x})\operatorname{cosec}^2\sqrt{x}}{2\sqrt{x}}$ 1

4. $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$ $\frac{1}{2}$

I.F. $= e^{\tan^{-1}y}$ $\frac{1}{2}$

SECTION B

5. A: card bears odd number
B: Number on the card is greater than 5 $\frac{1}{2}$

$A \cap B = \{7, 9, 11\}$ $\frac{1}{2}$

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{3/12}{7/12} = \frac{3}{7}$ $\frac{1}{2}+\frac{1}{2}$

6. Required probability $= \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4}$ 1

$= \frac{3}{7}$ 1

OR

$n = 5 \quad p = \frac{1}{3} \quad q = \frac{2}{3}$ $\frac{1}{2}$

$P(X \geq 4) = P(X = 4) + P(X = 5)$

$= 5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5$ 1

$= \frac{11}{243}$ $\frac{1}{2}$

7. $I = \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2(1+x^2)} dx$ 1

$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}[x - \tan^{-1}x] + C$ 1

OR

$I = \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\frac{5}{2} - 2x - x^2}}$ $\frac{1}{2}$

$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2 - (x+1)^2}}$ 1

$= \frac{1}{\sqrt{2}}\sin^{-1}\left[\frac{(x+1)\sqrt{2}}{\sqrt{7}}\right] + C$ $\frac{1}{2}$

8. $\int_{-\pi/4}^{0} \tan\left(\frac{\pi}{4} + x\right)dx = \log\left|\sec\left(\frac{\pi}{4} + x\right)\right|_{-\pi/4}^{0}$ $1+\frac{1}{2}$

$= \log\sqrt{2}$ or $\frac{1}{2}\log 2$ $\frac{1}{2}$

9. Position vector of z = $\frac{2(\vec{a} - 3\vec{b}) - 1(3\vec{a} + \vec{b})}{2-1}$ 1

$= -\vec{a} - 7\vec{b}$ 1

OR

$\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$ $\frac{1}{2}$

$\vec{a}-\vec{b}=-2\hat{i}+3\hat{j}-5\hat{k}$ $\frac{1}{2}$

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=-8+3+5=0$ $\frac{1}{2}$

so $(\vec{a}+\vec{b})\perp(\vec{a}-\vec{b})$ $\frac{1}{2}$

10. Given: $A' = A$, $B' = B$ $\frac{1}{2}$

$(AB - BA)' = (AB)' - (BA)'$ $\frac{1}{2}$

$= B'A' - A'B'$ $\frac{1}{2}$

$= BA - AB$

$= -(AB - BA)$, Hence $AB - BA$ is shew symmetric $\frac{1}{2}$

11. $a, b \in N \Rightarrow ab \in N \Rightarrow a + ab \in N$ 1

$\therefore$ * is binary

$a * (b * c) = a * (b + bc) = a + ab + abc$

$(a * b) * c = (a + ab) * c = a + ab + ac + abc$

In general $a * (b * c) \neq (a * b) * c$ $\therefore$ * is not associative 1

12. I.F. $= e^{\int 1\,dy} = e^{y}$ $\frac{1}{2}$

Solution is

$x\cdot e^{y} = \int e^{y}(\tan y + \sec^{2} y)dy + C$ 1

$x\cdot e^{y} = e^{y}\tan y + C$ or $x = \tan y + ce^{-y}$ $\frac{1}{2}$

SECTION C

13. Let $u = x^{\cos x}$, $v = (\cos x)^{\sin x}$

$y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ...(1) 1

$$\log u = \cos x \log x \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{\cos x}{x} - \sin x \log x$$

$$\frac{du}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right)$$ 1

$$\log v = \sin x \log \cos x \Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \tan x + \cos x \log \cos x$$

$$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$$ 1

So, $$\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right) + (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$$

14. Let $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

$$\text{RHS} = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$$ $\frac{1}{2}$

$$I = \int_0^1 x^2(1-x)^n\,dx$$

$$= \int_0^1 (1-x)^2 x^n dx$$ 1

$$= \int_0^1 [x^n + x^{n+2} - 2x^{n+1}]\,dx$$ $\frac{1}{2}$

$$= \left.\frac{x^{n+1}}{n+1} + \frac{x^{n+3}}{n+3} - \frac{2x^{n+2}}{n+2}\right|_0^1$$ 1

$$= \frac{1}{n+1} + \frac{1}{n+3} - \frac{2}{n+2}$$ $\frac{1}{2}$

15.
$$\left.\begin{aligned}\overrightarrow{BA} &= (x-4)\hat{i} - 6\hat{j} - 2\hat{k}\\ \overrightarrow{BC} &= -\hat{i} + 4\hat{j} + 3\hat{k}\\ \overrightarrow{BD} &= -8\hat{i} - \hat{j} + 3\hat{k}\end{aligned}\right\}$$ $1\frac{1}{2}$

As points are coplanar

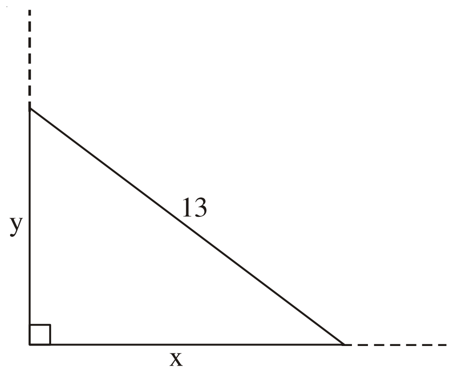
$$\therefore \begin{vmatrix} x-4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$ 1

$15(x - 4) + 6 \times 21 - 2 \times 33 = 0$ 1

$15x = 0$

$x = 0$ $\frac{1}{2}$

16. Figure $\frac{1}{2}$



$$\frac{dx}{dt} = 2\text{ cm/sec}$$ $\frac{1}{2}$

$$y = \sqrt{169 - x^2}$$ 1

$$\frac{dy}{dt} = -\frac{x}{\sqrt{169 - x^2}} \frac{dx}{dt}$$ 1

$$\left(\frac{dy}{dt}\right)_{x=5} = \frac{-5}{6}\text{ cm/sec}$$ 1

Hence height is decreasing at the rate $\frac{5}{6}$ cm / sec

17. Equation of required plane is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$ 2

$12(x - 3) - 16(y + 1) + 12(z - 2) = 0$

$3x - 4y + 3z = 19$

$\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 19$ 1

Distance from origin = $\frac{19}{\sqrt{34}}$ or $\frac{19\sqrt{34}}{34}$ 1

18. Given differential equation can be written as

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2) \quad 1$$

$$\int \frac{dy}{1+y^2} = \int (x^2 + 1)\, dx \quad 1$$

$$\tan^{-1} y = \frac{x^3}{3} + x + C \quad 1$$

$$x = 0,\ y = 1 \Rightarrow C = \frac{\pi}{4} \quad \frac{1}{2}$$

So particular solution is $\tan^{-1} y = \frac{x^3}{3} + x + \frac{\pi}{4}$ $\frac{1}{2}$

OR

Clearly given differential equation can be written as $\frac{dy}{dx} = \frac{y/x}{1+(y/x)^2}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

Given equation becomes

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$$

$$\int \frac{(1+v^2)}{v^3} dV = -\int \frac{dx}{x} \quad 1$$

$$-\frac{1}{2v^2} + \log|v| = -\log|x| + C \quad 1$$

$$-\frac{x^2}{2y^2} + \log|y| = C$$

$x = 0,\ y = 1 \Rightarrow C = 0$ $\frac{1}{2}$

So, particular solution is $\log|y| = \frac{x^2}{2y^2}$ $\frac{1}{2}$

19. Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$ $\frac{1}{2}$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$

$\Rightarrow x_1 = x_2$ 1

So f is one – one

For range let $f(x) = y$

$\frac{x - 1}{x - 2} = y$ $\frac{1}{2}$

$x = \frac{2y - 1}{y - 1}$ 1

Range of $f = R - \{1\}$ = co domain B $\frac{1}{2}$

So f is onto.

$f^{-1}(y) = \frac{2y - 1}{y - 1}$ or $f^{-1}(x) = \frac{2x - 1}{x - 1}$ $\frac{1}{2}$

OR

For reflexive 1

For symmetric 1

For transitive

Let $(a, b) \in S$ & $(b, c) \in S$

$|a - b| = 3m,\ |b - c| = 3n$

$a - b = \pm 3m \qquad b - c = \pm 3n$

$a - c = 3(\pm m \pm n) \Rightarrow a - c$ is divisible by 3 $1\frac{1}{2}$

$\Rightarrow |a - c|$ is divisible by 3

$\Rightarrow (a, c) \in S$

S is transitive

As S is reflexive, symmetric & transitive

$\therefore$ S is an equivalence reation. $\frac{1}{2}$

20. $\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = p \cos pt \Rightarrow \frac{dy}{dx} = \frac{p \cos pt}{\cos t}$ 1

$$\frac{dy}{dx} = \frac{p\sqrt{1-y^2}}{\sqrt{1-x^2}}$$ 1

$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = p^2(1-y^2)$ differentiating both sides w.r.t x $\frac{1}{2}$

$$\Rightarrow 2(1-x^2)\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -2p^2y\frac{dy}{dx}$$ 1

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$ $\frac{1}{2}$

OR

Let $\theta = \cos^{-1} x^2 \Rightarrow x^2 = \cos\theta$ 1

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}\right)$$ 1

$$= \frac{\pi}{4} - \frac{1}{2}\theta$$ 1

$$\therefore \frac{dy}{d\theta} = -\frac{1}{2}$$ 1

21. $R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 2x+2y & 2x+2y & 2x+2y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k(x^3+y^3)$$ 1

$$2(x+y)\begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = k(x^3+y^3)$$ $\frac{1}{2}$

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$$2(x+y)\begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix} = k(x^3+y^3)$$ $1\frac{1}{2}$

$2(x + y)(-x^2 + xy - y^2) = k(x^3 + y^3)$ $\frac{1}{2}$

$-2(x^3 + y^3) = k(x^3 + y^3)$

$\therefore\ k = -2$ $\frac{1}{2}$

22. $I = \dfrac{1}{\cos(b-a)} \displaystyle\int \frac{\cos[(x-a)-(x-b)]}{\sin(x-a)\cos(x-b)} dx$ 2

$$= \frac{1}{\cos(b-a)} \int \left[\frac{\cos(x-a)\ \cancel{\cos(x-b)}}{\sin(x-a)\ \cancel{\cos(x-b)}} + \frac{\sin\ \cancel{(x-a)}\ \sin(x-b)}{\sin\ \cancel{(x-a)}\ \cos(x-b)} \right] dx$$ 1

$$= \frac{1}{\cos(b-a)} [\log|\sin(x-a)| + \log|\sec(x-b)|] + C$$ 1

23. L.H.S. becomes

$$\tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$$ $\frac{1}{2}+\frac{1}{2}$

$$= \tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}} \right) = \tan^{-1}\frac{77}{36}$$ 1+1

$$= \cot^{-1}\frac{36}{77} = \text{RHS}$$ 1

SECTION D

24. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ we know that A = IA

i.e., $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 14 & -25 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_2 \to -(R_2 - 3R_3)$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 1 & -2 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + 4R_2$

$R_3 \to R_3 - 5R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 11 \\ 0 & -1 & 3 \\ -1 & 5 & -13 \end{bmatrix} A$$ 4

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} A$$

$\Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ 1

OR

$|A| = 67 \neq 0 \qquad \therefore\ X = A^{-1}B$ $1+\frac{1}{2}$

$$\text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ 2

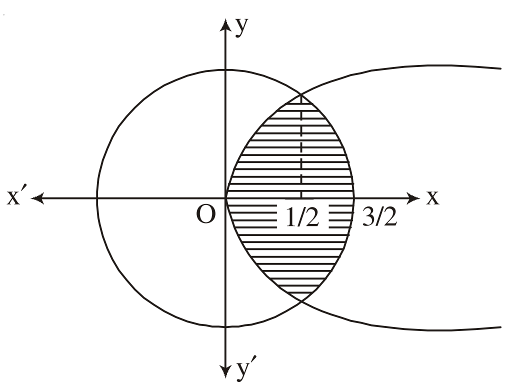
$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ $\frac{1}{2}$

$$\text{So } X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$ $1\frac{1}{2}$

$x = 3,\ y = -2,\ z = 1$ $\frac{1}{2}$

25. Correct Figure 1



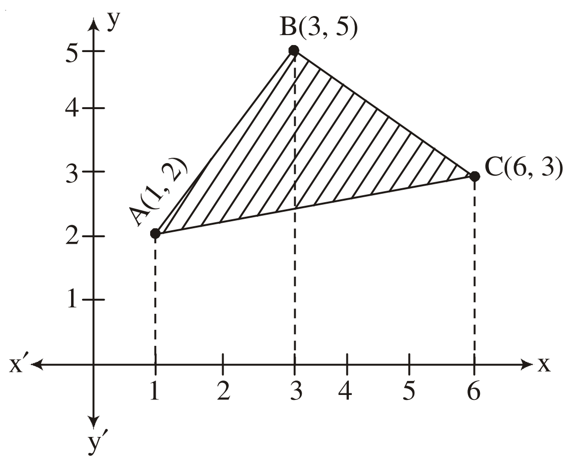
x coordinate of Point of intersection $= \frac{1}{2}$ 1

$$\text{Required Area} = 2\left[\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{3/2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2}\,dx\right]$$ 2

$$= 2\left[\frac{4}{3}x^{3/2}\Big|_0^{\frac{1}{2}} + \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Big|_{\frac{1}{2}}^{3/2}\right]$$ 1

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$ 1

OR



Correct figure 1

Correct points of intersection $1\frac{1}{2}$

Required Area $= \int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{21-2x}{3}dx - \int_1^6 \frac{x+9}{5}dx$ 2

$= \left(\frac{3x^2}{4}+\frac{x}{2}\right)\Big|_1^3 + \left(7x-\frac{x^2}{3}\right)\Big|_3^6 - \left(\frac{x^2}{10}+\frac{9x}{5}\right)\Big|_1^6$ 1

$= 7+12-\frac{25}{2}$

$= \frac{13}{2}$ $\frac{1}{2}$

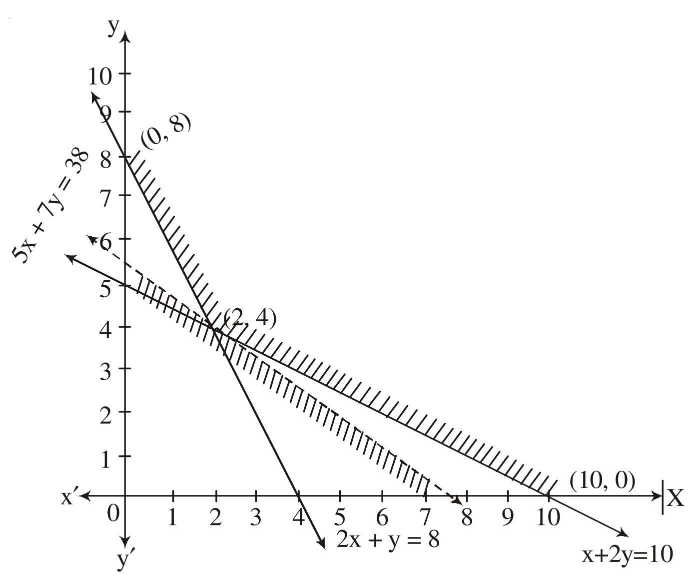
26. Let Quantity of Food I = x kg

Quantity of Food II = y kg

Out L.P.P. is

Minimize Z = 50x + 70y subject to $\frac{1}{2}$

$$\left.\begin{aligned} 2x + y &\geq 8 \\ x + 2y &\geq 10 \\ x \geq 0,\ y &\geq 0 \end{aligned}\right\}$$ $2\frac{1}{2}$



Correct Graph 2

Corner points	Value of Z		
(10, 0)	500		
(2, 4)	380	$\rightarrow$ minimum	$\frac{1}{2}$
(0, 8)	560		

Consider 50x + 70y < 380

which has no point common with feasible region

So minimum value of Z = ₹ 380 $\frac{1}{2}$

at x = 2 kg, y = 4 kg

27. Required equation of line is

$$\vec{r} = 2\hat{i} + 3\hat{j} + 2\hat{k} + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$$ 2

$$\vec{a}_1 = -2\hat{i} + 3\hat{j}, \vec{a}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}, \quad \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 2\hat{k}$$ 1

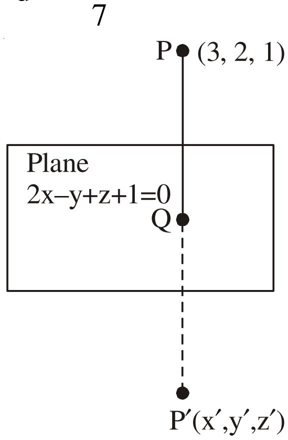
$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix}$$

$$= 6\hat{i} - 20\hat{j} - 12\hat{k}$$ 2

$$d = \frac{\sqrt{580}}{7}$$ 1

OR



Correct figure 1

Equation of PQ

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda$$ $\frac{1}{2}$

Coodinates of Q $(2\lambda + 3, -\lambda + 2, \lambda + 1)$ $\frac{1}{2}$

As Q lies on plane

$\therefore\ 4\lambda + 6 + \lambda - 2 + \lambda + 1 = -1$

gives, $\lambda = -1$ 1

Coordinates of Q (1, 3, 0) $\frac{1}{2}$

$PQ = \sqrt{6}$ 1

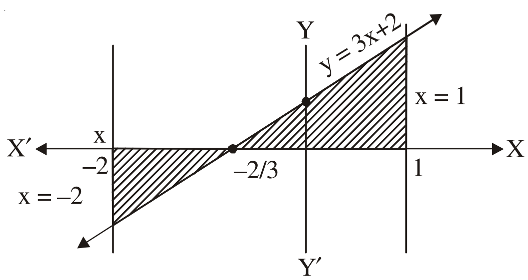
For unique $P'(x', y', z')$

$\frac{x'+3}{2} = 1,\ \frac{y'+2}{2} = 3,\ \frac{z'+1}{2} = 0$ 1

$x' = -1\ y' = 4\ z' = -1$

image is (–1, 4, –1) $\frac{1}{2}$

28. Correct figure 1



Required Area $= \left|\int_{-2}^{-2/3}(3x+2)\,dx\right| + \int_{-\frac{2}{3}}^{1}(3x+2)\,dx$ 3

$= \left|\frac{+(3x+2)^2}{6}\Big|_{-2}^{\frac{-2}{3}} + \frac{(3x+2)^2}{6}\right|\Big|_{\frac{-2}{3}}^{1}$ 1

$= \frac{8}{3} + \frac{25}{6}$

$= \frac{41}{6}$ 1

29. E_1: Selected coin has tail on both faces

E_2: Selected coin is biased

E_3: Selected coin is unbiased 1

A: Tail comes up

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ 1

$P(A|E_1) = 1,\ P(A|E_2) = \frac{7}{10},\ P(A|E_3) = \frac{1}{2}$ $1\frac{1}{2}$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{7}{10} + \frac{1}{3}\times\frac{1}{2}}$$ 2

$= \frac{10}{22}$ or $\frac{5}{11}$ $\frac{1}{2}$