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Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/4/1, 65/4/2, 65/4/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/4/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $xy = A + 5x \Rightarrow x\frac{dy}{dx} + y = 5$ $\qquad \frac{1}{2}+\frac{1}{2}$

2. $|2\text{ adj }A| = 2^3\,|A|^{3-1} = 8 \times 81 = 648$ $\qquad \frac{1}{2}+\frac{1}{2}$

3. $\cos\theta = \frac{3+12-4}{3\times7} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\right)$ $\qquad \frac{1}{2}+\frac{1}{2}$

OR

$\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1$, length of $x-$intercept $=\frac{5}{2}$ $\qquad 1$

4. $\frac{dy}{dx} = \frac{-\sin e^x}{\cos e^x}\cdot e^x \text{ or } -e^x\cdot\tan e^x$ $\qquad 1$

SECTION B

5. $\int_{-\pi/4}^{0} \tan\left(\frac{\pi}{4}+x\right)dx = \log\left|\sec\left(\frac{\pi}{4}+x\right)\right|_{-\pi/4}^{0}$ $\qquad 1+\frac{1}{2}$

$= \log\sqrt{2} \text{ or } \frac{1}{2}\log 2$ $\qquad \frac{1}{2}$

6. $a * b = 2a + b \in R \quad \forall\, a, b \in R$ $\qquad \therefore$ * is binary $\qquad 1$

$a * (b * c) = a * (2b + c) = 2a + 2b + c$

$(a * b) * c = (2a + b) * c = 4a + 2b + c$

In genral $a * (b * c) \neq (a * b) * c$ $\qquad \therefore$ * is not associative $\qquad 1$

7. Position vector of z = $\frac{2(\vec{a}-3\vec{b})-1(3\vec{a}+\vec{b})}{2-1}$ $\qquad 1$

$= -\vec{a}-7\vec{b}$ $\qquad 1$

OR

$\vec{a}+\vec{b} = 4\hat{i}+\hat{j}-\hat{k}$ $\qquad \frac{1}{2}$

$\vec{a}-\vec{b} = -2\hat{i}+3\hat{j}-5\hat{k}$ $\qquad \frac{1}{2}$

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = -8+3+5 = 0$ $\frac{1}{2}$

so $(\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$ $\frac{1}{2}$

8. Given: A′ = A, B′ = B $\frac{1}{2}$

(AB – BA)′ = (AB)′ – (BA)′ $\frac{1}{2}$

= B′A′ – A′B′ $\frac{1}{2}$

= BA – AB

= –(AB – BA), Hence AB – BA is shew symmetric $\frac{1}{2}$

9. A: card bears odd number
B: Number on the card is greater than 5 $\frac{1}{2}$

$A \cap B = \{7, 9, 11\}$ $\frac{1}{2}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/12}{7/2} = \frac{3}{7}$ $\frac{1}{2}+\frac{1}{2}$

10. Required probability $= \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4}$ 1

$= \frac{3}{7}$ 1

OR

$n = 5 \quad p = \frac{1}{3} \quad q = \frac{2}{3}$ $\frac{1}{2}$

$P(X \geq 4) = P(X = 4) + P(X = 5)$

$= 5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5$ 1

$= \frac{11}{243}$ $\frac{1}{2}$

11. I.F. $= e^x$ $\frac{1}{2}$

Solution is $y.e^x = \int (\cos x - \sin x) e^x \, dx + C$ $\frac{1}{2}$

$y.e^x = e^x \cos x + C$ or $y = \cos x + Ce^{-x}$ 1

12. $I = \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2(1+x^2)} dx$ 1

$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}[x - \tan^{-1} x] + C$ 1

OR

$I = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - 2x - x^2}}$ $\frac{1}{2}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2 - (x+1)^2}}$ 1

$= \frac{1}{\sqrt{2}} \sin^{-1} \left[\frac{(x+1)\sqrt{2}}{\sqrt{7}} \right] + C$ $\frac{1}{2}$

SECTION C

13. $R_1 \to R_1 + R_2 + R_3$ & taking $12 + x$ common

$$(12+x) \begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$ $1+\frac{1}{2}$

$C_2 \to C_2 - C_1 \quad C_3 \to C_3 - C_1$

$$(12+x) \begin{vmatrix} 1 & 0 & 0 \\ 4+x & -2x & 0 \\ 4+x & 0 & -2x \end{vmatrix} = 0$$ $1\frac{1}{2}$

$4x^2 (12 + x) = 0$ $\frac{1}{2}$

$x = 0$ or $x = -12$ $\frac{1}{2}$

14. Given differential equation can be written as

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$ 1

$$\int \frac{dy}{1+y^2} = \int (x^2+1)\,dx$$ 1

$$\tan^{-1} y = \frac{x^3}{3} + x + C$$ 1

$x = 0,\ y = 1 \Rightarrow C = \frac{\pi}{4}$ $\frac{1}{2}$

So particular solution is $\tan^{-1} y = \frac{x^3}{3} + x + \frac{\pi}{4}$ $\frac{1}{2}$

OR

Clearly given differential equation can be written as $\frac{dy}{dx} = \frac{y/x}{1+(y/x)^2}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

Given equation becomes

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$$

$$\int \frac{(1+v^2)}{v^3} dV = -\int \frac{dx}{x}$$ 1

$$-\frac{1}{2v^2} + \log|v| = -\log|x| + C$$ 1

$$-\frac{x^2}{2y^2} + \log|y| = C$$

$x = 0,\ y = 1 \Rightarrow C = 0$ $\frac{1}{2}$

So, particular solution is $\log|y| = \frac{x^2}{2y^2}$ $\frac{1}{2}$

15. Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$ $\frac{1}{2}$

$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$

$\Rightarrow x_1 = x_2$ 1

So f is one – one

For range let f(x) = y

$\frac{x - 1}{x - 2} = y$ $\frac{1}{2}$

$x = \frac{2y - 1}{y - 1}$ 1

Range of f = R – {1} = co domain B $\frac{1}{2}$

So f is onto.

$f^{-1}(y) = \frac{2y - 1}{y - 1}$ or $f^{-1}(x) = \frac{2x - 1}{x - 1}$ $\frac{1}{2}$

OR

For reflexive 1

For symmetric 1

For transitive

Let (a, b) $\in$ S & (b, c) $\in$ S

$|a - b| = 3m$, $|b - c| = 3n$

$a - b = \pm 3m$ $\qquad$ $b - c = \pm 3n$

$a - c = 3(\pm m \pm n) \Rightarrow a - c$ is divisible by 3

$\Rightarrow |a - c|$ is divisible by 3

$\Rightarrow (a, c) \in S$

S is transitive $\quad 1\frac{1}{2}$

As S is reflexive, symmetric & transitive

$\therefore$ S is an equivalence reation. $\quad \frac{1}{2}$

16. $I = \int \frac{\cos(x + b + a - b)}{\sin(x + b)} dx$ $\quad 1$

$= \int [\cot(x + b)\cos(a - b) - \sin(a - b)]\,dx$ $\quad 1$

$= \cos(a - b)\log|\sin(x + b)| - x\sin(a - b) + C$ $\quad 2$

17. $\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = p\cos pt \;\Rightarrow\; \frac{dy}{dx} = \frac{p\cos pt}{\cos t}$ $\quad 1$

$\frac{dy}{dx} = \frac{p\sqrt{1 - y^2}}{\sqrt{1 - x^2}}$ $\quad 1$

$\Rightarrow (1 - x^2)\left(\frac{dy}{dx}\right)^2 = p^2(1 - y^2)$ differentiating both sides w.r.t x $\quad \frac{1}{2}$

$\Rightarrow 2(1 - x^2)\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = -2p^2y\frac{dy}{dx}$ $\quad 1$

$\Rightarrow (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$ $\quad \frac{1}{2}$

OR

Let $\theta = \cos^{-1} x^2 \Rightarrow x^2 = \cos\theta$ $\quad 1$

$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1 + \cos\theta} - \sqrt{1 - \cos\theta}}{\sqrt{1 + \cos\theta} + \sqrt{1 - \cos\theta}}\right)$

$$= \tan^{-1}\left(\frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}\right)$$ 1

$$= \frac{\pi}{4} - \frac{1}{2}\theta$$ 1

$$\therefore \quad \frac{dy}{d\theta} = -\frac{1}{2}$$ 1

18. LHS becomes $\tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$ 1

$$= \tan^{-1}\left(\frac{\frac{5}{12} + \frac{3}{4}}{1 - \frac{5}{12}\cdot\frac{3}{4}}\right) = \tan^{-1}\frac{56}{33}$$ 1+1

$$= \sin^{-1}\frac{56}{65} = \text{RHS}$$ 1

19. Let $u = x^{\cos x}$, $v = (\cos x)^{\sin x}$

$y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$ 1

$$\log u = \cos x \log x \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{\cos x}{x} - \sin x \log x$$

$$\frac{du}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right)$$ 1

$$\log v = \sin x \log \cos x \Rightarrow \frac{1}{v}\frac{dv}{dx} = -\sin x \tan x + \cos x \log \cos x$$

$$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$$ 1

$$\text{So, } \frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \log x\right) + (\cos x)^{\sin x}(-\sin x \tan x + \cos x \log \cos x)$$ 1

20. Let $a - x = t \Rightarrow -dx = dt$ $\frac{1}{2}$

$$\text{RHS} = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$$ $\frac{1}{2}$

$$I = \int_0^1 x^2(1-x)^n\,dx$$

$$= \int_0^1 (1-x)^2 x^n dx$$ 1

$$= \int_0^1 [x^n + x^{n+2} - 2x^{n+1}]\,dx$$ $\frac{1}{2}$

$$= \left.\frac{x^{n+1}}{n+1} + \frac{x^{n+3}}{n+3} - \frac{2x^{n+2}}{n+2}\right|_0^1$$ 1

$$= \frac{1}{n+1} + \frac{1}{n+3} - \frac{2}{n+2}$$ $\frac{1}{2}$

21.

$$\left.\begin{aligned} \overrightarrow{BA} &= (x-4)\hat{i} - 6\hat{j} - 2\hat{k} \\ \overrightarrow{BC} &= -\hat{i} + 4\hat{j} + 3\hat{k} \\ \overrightarrow{BD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\}$$ $1\frac{1}{2}$

As points are coplanar

$$\therefore \begin{vmatrix} x-4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$ 1

$15(x - 4) + 6 \times 21 - 2 \times 33 = 0$ 1

$15x = 0$

$x = 0$ $\frac{1}{2}$

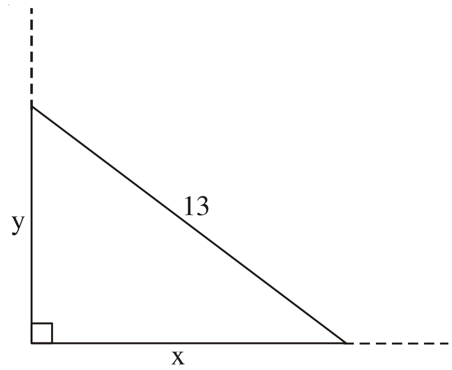
22. 

Figure $\frac{1}{2}$

$\frac{dx}{dt} = 2 \text{ cm/sec}$ $\frac{1}{2}$

$y = \sqrt{169 - x^2}$ 1

$\frac{dy}{dt} = -\frac{x}{\sqrt{169 - x^2}} \frac{dx}{dt}$ 1

$\left(\frac{dy}{dt}\right)_{x=5} = \frac{-5}{6} \text{ cm/sec}$ 1

Hence height is decreasing at the rate $\frac{5}{6}$ cm / sec

23. Equation of required plane is

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$ 2

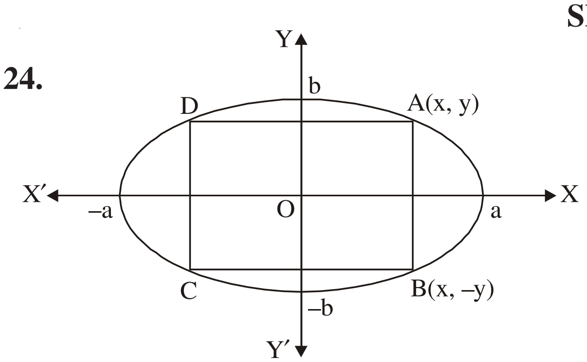
$12(x - 3) - 16(y + 1) + 12(z - 2) = 0$

$3x - 4y + 3z = 19$

$\vec{r} \cdot (3\hat{i} - 4\hat{j} + 3\hat{k}) = 19$ 1

Distance from origin = $\frac{19}{\sqrt{34}}$ or $\frac{19\sqrt{34}}{34}$ 1

SECTION D

24. 

Correct Figure 1

Let ABCD is a rectangle

A = 4xy

$A^2 = 16x^2(a^2 - x^2)\frac{b^2}{a^2} = f(x)$ 1

$f'(x) = \frac{16b^2}{a^2}(2a^2 x - 4x^3)$ 1

$$f'(x) = 0 \Rightarrow x = \frac{a}{\sqrt{2}} \Rightarrow y = \frac{b}{\sqrt{2}}$$ 1

$$f''(x) = \frac{16b^2}{a^2}(2a^2 - 12x^2) = \frac{16b^2}{a^2}(-4a^2) < 0 \text{ at } x = \frac{a}{\sqrt{2}}$$

Area is maximum at $x = \frac{a}{\sqrt{2}}, y = \frac{b}{\sqrt{2}}$ 1

Required Area = $4\int_0^{a/\sqrt{2}} \frac{b}{\sqrt{2}}dx$ $\frac{1}{2}$

$= 4\frac{b}{\sqrt{2}}\frac{a}{\sqrt{2}} = 2ab$ $\frac{1}{2}$

Note: Since finding maximum/minimum area using integration is not discussed in prescribed books, so a student who tried to attempt but could not complete may be given full marks.

25. E_1: Selected person is cyclist
E_2: Selected person is scooter driver
E_3: Selected person is car driver
A: insured person met with an accident } 1

$P(E_1) = \frac{3}{18}, \ P(E_2) = \frac{6}{18}, \ P(E_3) = \frac{9}{18}$
$P(A|E_1) = 0.3, P(A|E_2) = 0.05, P(A|E_3) = 0.02$ } 2

$$P(E_1/A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3)}$$

$$= \frac{\frac{3}{18}\times\frac{30}{100}}{\frac{3}{18}\times\frac{30}{100} + \frac{6}{18}\times\frac{5}{100} + \frac{9}{18}\times\frac{2}{100}}$$ 2

$$= \frac{90}{138} \text{ or } \frac{15}{23}$$ 1

26. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ we know that A = IA i.e., $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & -4 & 7 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_1, R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 14 & -25 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_2 \to -(R_2 - 3R_3)$

$$\begin{bmatrix} 1 & -4 & 7 \\ 0 & 1 & -2 \\ 0 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} A$$

$R_1 \to R_1 + 4R_2$

$R_3 \to R_3 - 5R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 11 \\ 0 & -1 & 3 \\ -1 & 5 & -13 \end{bmatrix} A$$ 4

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} A$$

$\Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & -4 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ 1

OR

$|A| = 67 \neq 0 \qquad \therefore X = A^{-1}B$ $1+\frac{1}{2}$

adj $A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ 2

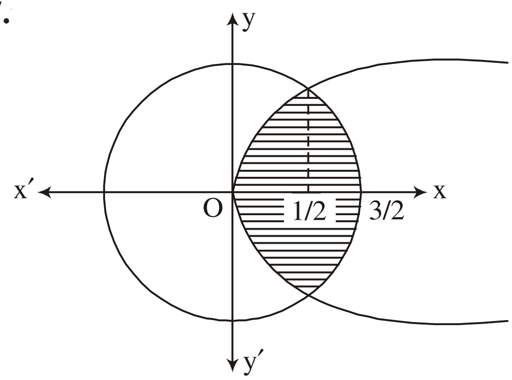
$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ $\frac{1}{2}$

So $$X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$ $1\frac{1}{2}$

$x = 3,\ y = -2,\ z = 1$ $\frac{1}{2}$

27.



Correct Figure 1

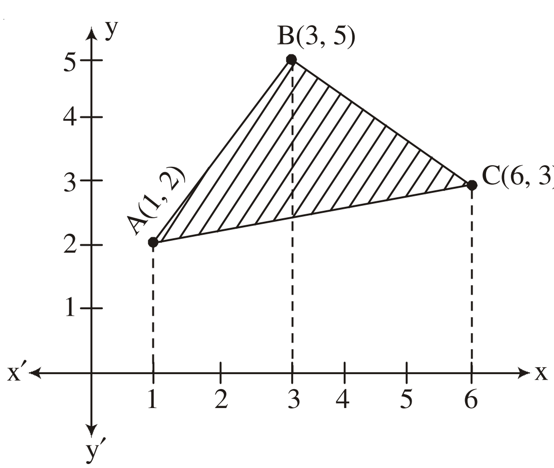
x coordinate of Point of intersection = $\frac{1}{2}$ 1

Required Area = $2\left[\int_0^{\frac{1}{2}} 2\sqrt{x}\,dx + \int_{\frac{1}{2}}^{3/2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2}\,dx\right]$ 2

$= 2\left[\frac{4}{3}x^{3/2}\Big|_0^{\frac{1}{2}} + \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Big|_{\frac{1}{2}}^{3/2}\right]$ 1

$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$ 1

OR



Correct figure 1

Correct points of intersection $1\frac{1}{2}$

Required Area = $\int_1^3 \frac{3x+1}{2}dx + \int_3^6 \frac{21-2x}{3}dx - \int_1^6 \frac{x+9}{5}dx$ 2

$$= \left(\frac{3x^2}{4} + \frac{x}{2}\right)\Bigg|_1^3 + \left(7x - \frac{x^2}{3}\right)\Bigg|_3^6 - \left(\frac{x^2}{10} + \frac{9x}{5}\right)\Bigg|_1^6$$ 1

$$= 7 + 12 - \frac{25}{2}$$

$$= \frac{13}{2}$$ $\frac{1}{2}$

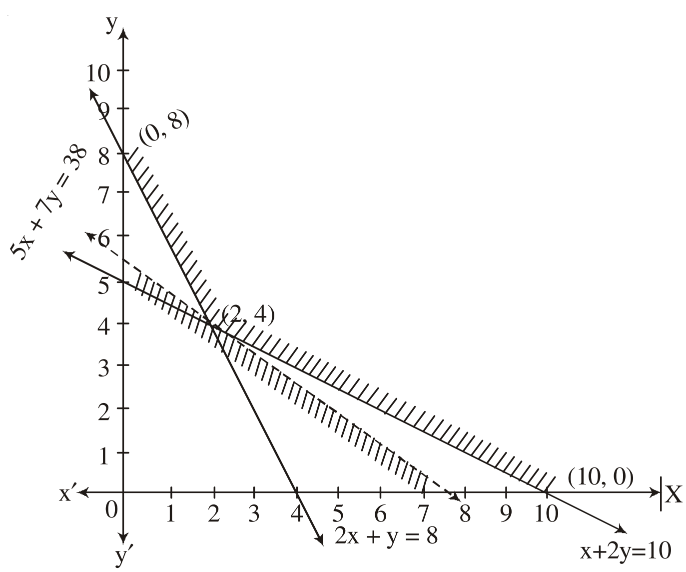
28. Let Quantity of Food I = x kg

Quantity of Food II = y kg

Out L.P.P. is

Minimize Z = 50x + 70y subject to $\frac{1}{2}$

$$\left.\begin{array}{l} 2x + y \geq 8 \\ x + 2y \geq 10 \\ x \geq 0,\ y \geq 0 \end{array}\right\}$$ $2\frac{1}{2}$



Correct graph 2

Corner points	**Value of Z**	
(10, 0)	500	
(2, 4)	380	$\rightarrow$ minimum
(0, 8)	560	

$\frac{1}{2}$

Consider 50x + 70y < 380

which has no point common with feasible region

So minimum value of Z = ₹ 380 $\frac{1}{2}$

at x = 2 kg, y = 4 kg

29. Required equation of line is

$\vec{r} = 2\hat{i} + 3\hat{j} + 2\hat{k} + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$ 2

$\vec{a}_1 = -2\hat{i} + 3\hat{j}, \vec{a}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}, \quad \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 2\hat{k}$ 1

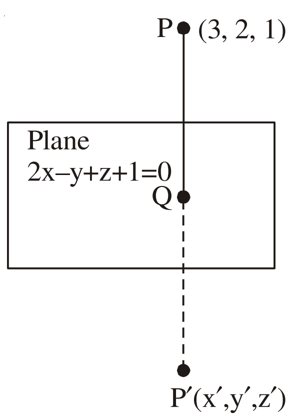
$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix}$$

$= 6\hat{i} - 20\hat{j} - 12\hat{k}$ 2

$d = \frac{\sqrt{580}}{7}$ 1

OR



Correct figure 1

Equation of PQ

$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda$ $\frac{1}{2}$

Coodinates of Q $(2\lambda + 3, -\lambda + 2, \lambda + 1)$ $\frac{1}{2}$

As Q lies on plane

$\therefore\ 4\lambda + 6 + \lambda - 2 + \lambda + 1 = -1$

gives, $\lambda = -1$ 1

Coordinates of Q (1, 3, 0) $\frac{1}{2}$

$PQ = \sqrt{6}$ 1

For unique $P'(x', y', z')$

$\frac{x'+3}{2} = 1, \ \frac{y'+2}{2} = 3, \ \frac{z'+1}{2} = 0$ 1

$x' = -1 \ y' = 4 \ z' = -1$

image is (–1, 4, –1) $\frac{1}{2}$